Turbulence Environnementale
M2R DET parcours MFE

Chapitre 4 : Turbulence, rotation
et stratification

I- Turbulence à très faible Rossby : la turbulence 2D

Lien avec la turbulence ???

Sur plan β ou sur plan f, les grandes échelles vérifient cette équation :

$$J\left(\psi, -\Delta\psi + \beta y - \frac{1}{L_D^2}\psi\right) = 0$$

Pour des échelles plus petites que L_{De} ? L << L_{De} ?
→ Approximation du toit rigide donne la même chose, à savoir :
$J(\psi, -\Delta\psi+\beta y)=0$
Sur plan f : $J(\psi, -\Delta\psi)=0$!!! Purement 2D !!! Turbulence 2D !!!

Turbulence 2D

$$\frac{\partial \nabla^2\psi}{\partial t} + J(\psi, \nabla^2\psi) = F + \nu\nabla^4\psi.$$ avec vorticité $\zeta=\Delta\psi$

Principal souci : plus d'étirement-basculement….
et donc ? Plus de cascade au sens de Kolmogorov…

I- Turbulence à très faible Rossby : la turbulence 2D

avec ν=0 et F=0 :

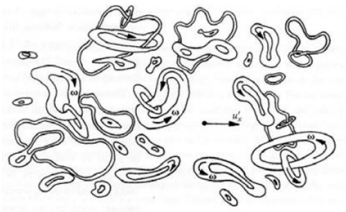
$$\hat{E} = \frac{1}{2}\int_A (u^2 + v^2)\,\mathrm{d}A = \frac{1}{2}\int_A (\nabla\psi)^2\,\mathrm{d}A,$$

$$\hat{Z} = \frac{1}{2}\int_A \zeta^2\,\mathrm{d}A = \frac{1}{2}\int_A (\nabla^2\psi)^2\,\mathrm{d}A,$$

$$\frac{\mathrm{d}\hat{E}}{\mathrm{d}t} = -\nu\int \zeta^2\,\mathrm{d}A.$$

$$\frac{\mathrm{d}\hat{Z}}{\mathrm{d}t} = 0,$$

Turbulence 3D :



Pas de limite pour $\hat{Z} = \int \zeta^2\,\mathrm{dV}$ du fait de l'étirement…
Donc dE/dt=-ε

Turbulence 2D :

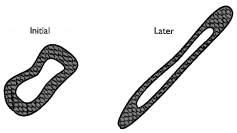


Fig. 8.5 In incompressible two-dimensional flow, a band of fluid will generally be elongated, but its area is preserved. Since vorticity is tied to fluid parcels, the values of the vorticity in the shaded area (and in the hole in the middle) are maintained; thus, vorticity gradients will increase and the enstrophy is thereby, on average, moved to smaller scales.

Pas d'étirement, d'où $\hat{Z}$
Ne peut que diminuer, donc
Dans la limite ν→0, E≈C_{ste}

I- Turbulence à très faible Rossby : la turbulence 2D

Conséquence sur le spectre d'énergie

Cascade inverse d'énergie vers les grandes longueurs d'onde (demo Vallis p352)

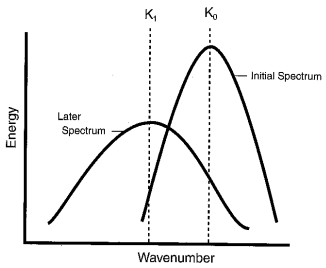
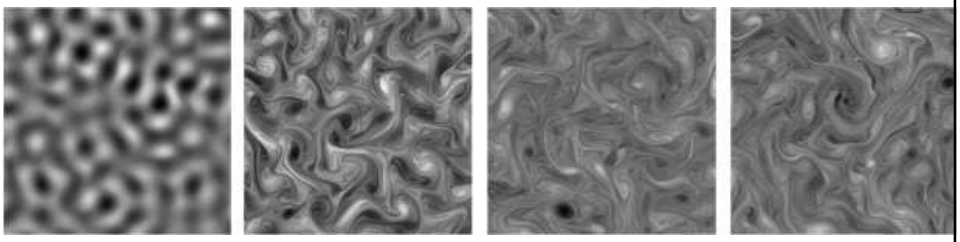


Fig. 8.6 In two-dimensional flow, the centroid of the energy spectrum will move to large scales (smaller wavenumber) provided that the width of the distribution increases — as can be expected in a nonlinear, eddying flow.



I- Turbulence à très faible Rossby : la turbulence 2D

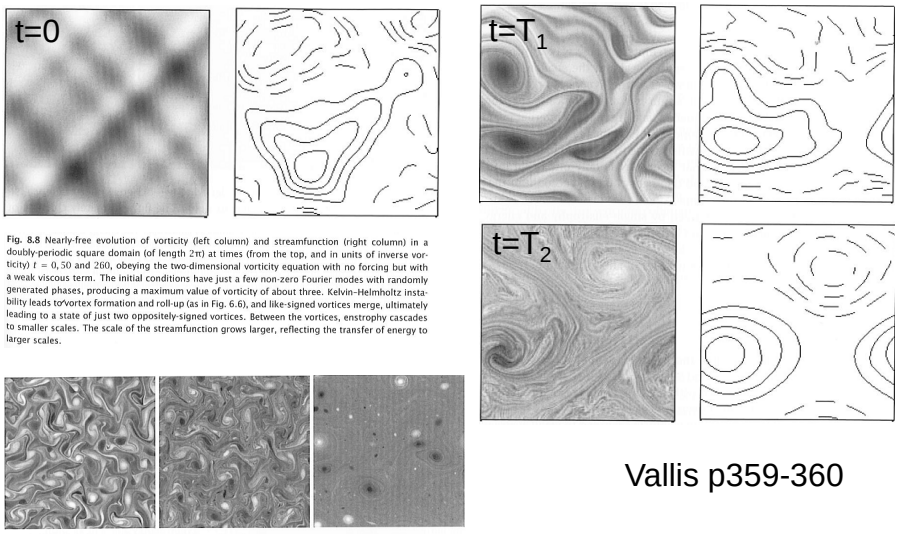


Fig. 8.8 Nearly-free evolution of vorticity (left column) and streamfunction (right column) in a doubly-periodic square domain (of length 2π) at times (from the top, and in units of inverse vorticity) $t = 0, 50$ and 260, obeying the two-dimensional vorticity equation with no forcing but with a weak viscous term. The initial conditions have just a few non-zero Fourier modes with randomly generated phases, producing a maximum value of vorticity of about three. Kelvin–Helmholtz instability leads to vortex formation and roll-up (as in Fig. 6.6), and like-signed vortices merge, ultimately leading to a state of just two oppositely-signed vortices. Between the vortices, enstrophy cascades to smaller scales. The scale of the streamfunction grows larger, reflecting the transfer of energy to larger scales.

Fig. 8.9 Snapshots of the vorticity field in decaying two-dimensional turbulence, similar to Fig. 8.8, with time increasing left to right. The flow ultimately consists of a small number of vortices whose trajectories are similar to that of interacting point vortices, with occasional close encounters leading to vortex merger.

Vallis p359-360

I- Turbulence à très faible Rossby : la turbulence 2D

Turbulence 2D forcée à l'échelle k_f :

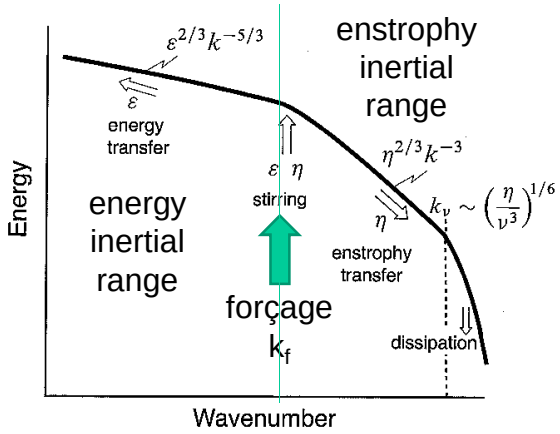


Fig. 8.7 The energy spectrum of two-dimensional turbulence. (Compare with Fig. 8.3.) Energy supplied at some rate ε is transferred to large scales, whereas enstrophy supplied at some rate η is transferred to small scales, where it may be dissipated by viscosity. If the forcing is localized at a scale k_f^{-1} then $\eta \approx k_f^2 \varepsilon$.

ε ici le taux de transfert d'énergie vers les grandes échelles
η ici le taux de transfert de l'enstrophie vers les petites échelles
Quand en voit-on la fin ? Limitation de l'énergie à grande échelle

I- Turbulence à très faible Rossby : effet de plan beta

Blocage par les ondes de Rossby

Vallis p381-382

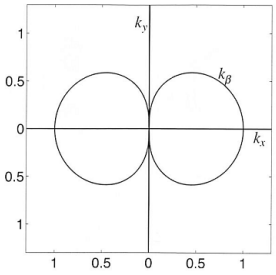


Fig. 9.2 The anisotropic wave–turbulence boundary k_β, in wave-vector space calculated by equating the turbulent eddy transfer rate, proportional to $k^{2/3}$ in a $k^{-5/3}$ spectrum, to the Rossby wave frequency $\beta k^x/k^2$, as in (9.14). Within the dumbbell Rossby waves dominate and energy transfer is inhibited. The inverse cascade plus Rossby waves thus leads to a generation of zonal flow.

Effet sur les spectres de turbulence

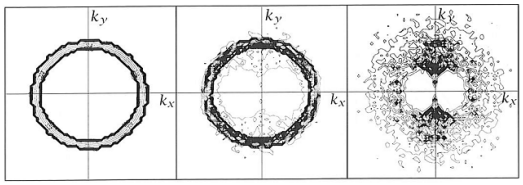


Fig. 9.3 Evolution of the energy spectrum in a freely evolving two-dimensional simulation on the β-plane. The panels show contours of energy in wavenumber (k_x, k_y) space at successive times. The initial spectrum is isotropic. The energy 'implodes', but its passage to large scales is impeded by the β-effect, and second and third panels show the spectrum at later times, illustrating the dumbbell predicted by (9.14) and Fig. 9.2.[2]

I- Turbulence à très faible Rossby : effet de plan beta

Equivalent dans le domaine spatial : Vallis p383-384

Fig. 9.4 Evolution of vorticity (greyscale, left column) and streamfunction (contour plots, right column) in a doubly periodic square domain (of length 2π) at times $t = 0, 50$ and 260 (in units of inverse vorticity), obeying (9.4) with the addition of a weak viscous term on the right-hand side. The initial conditions are the same as for Fig. 8.8, with a maximum value of vorticity of about 3. As $\beta = 3$, the β-Rossby number, $|\zeta|/\beta L$ is about unity. Compared to Fig. 8.8, vortex formation is inhibited and there is a tendency toward zonal flow.

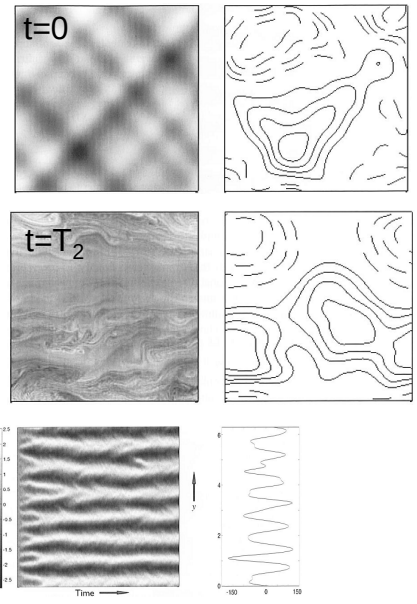


Fig. 9.5 Left: Grey-scale image of zonally averaged zonal velocity ($\overline{u}$) as a function of time and latitude (Y), produced in a simulation forced around wavenumber 80 and with $k_\beta = \sqrt{\beta/U} \approx 10$ (in a square domain of side 2π). Right: Values of $\partial^2\overline{u}/\partial y^2$ as a function of latitude, late in the integration. Jets form very quickly from the random initial conditions, and are subsequently quite steady.[4]

II- Turbulence en stratifié

The **Omidov scale** is defined as $L_o = \dfrac{\varepsilon^{1/2}}{N^{3/2}}$

It is a measure of the vertical scale of the largest eddies which may **overturn** in a stable stratification. It is larger than the Kolmogorov scale and does not depend on viscosity.

Numerical simulations (density fluctuations) of strongly stratified turbulence (Brethouver at al. 2007)

➡ La turbulence classique de Kolmogorov s'arrête quand L_0 devient plus petite que les échelles horizontales intégrales

$\leftrightarrow Fr_h<1$

Au-delà ? La stratification commence à jouer…pancake

I- Turbulence à très faible Rossby : la turbulence 2D

Type de turbulence ???? Dépend de Re et Fr_h

Take

l_v = typical vertical scale of the flow
l_h = typical horizontal scale of the flow
$\alpha = l_v/l_h$

Traditional argument:

l_v is imposed by external factors such that

$F_v = u/l_v N \ll 1$, ie $l_v \ll u/N$

Arguments of Brethouver et al 2007:

l_v is a free parameter

$\mathrm{Re} = u l_h/\nu \gg 1$

$F_h = u/N l_h \ll 1$

Strongly stratified regime if $R_b \gg \mathrm{Re}\, F_h^2$
which implies negligible viscous and diffusive effects

$\Rightarrow F_v \sim 1$, ie $u/N \sim l_v$

$\Rightarrow \alpha \sim F_h$

By definition of R_b and F_h and with $\varepsilon = u^3/l_h$

$F_h = \dfrac{\varepsilon}{Nu^2}$, $R_b = \dfrac{\varepsilon}{\nu N^2}$

$\dfrac{l_h}{L_o} = F_h^{-3/2}$, $\dfrac{l_v}{L_o} = F_h^{-1/2}$

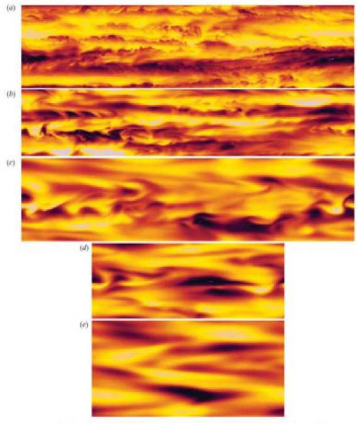
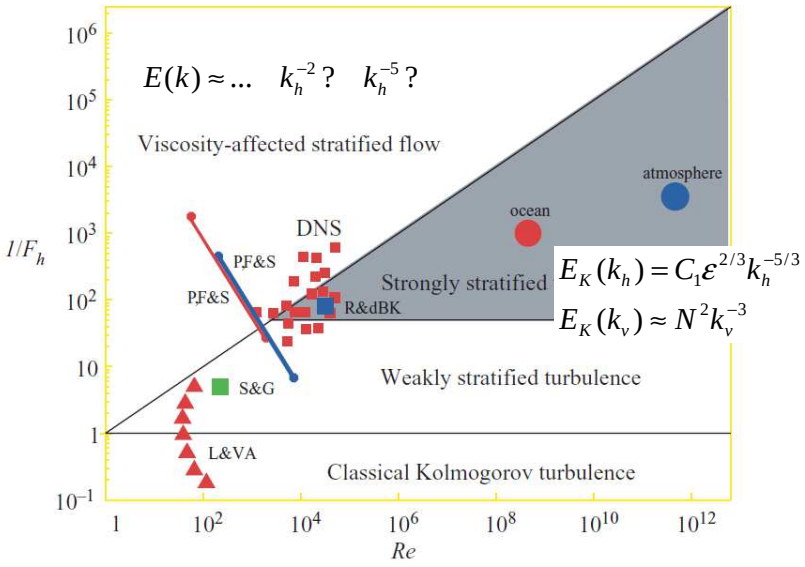


FIGURE 5. Snapshots of the density fluctuations in a vertical plane for $F_h \simeq 0.015$. (a) run D9.6, (b) B3.0, (c) A1.8, (d) R0.7 and (e) R0.3.

Run	Re ($\times 10^3$)	F_h ($\times 10^{-2}$)	$\mathscr{R}$	$\frac{L_h}{L_v}$	$N_h \times N_v$	$k_{max}\eta$	$\frac{l_h}{L_h}$	$\frac{l_v}{l_h}$	$\frac{\langle u_z^2\rangle}{2E_K}$	$\frac{-\langle u_z'\rho'\rangle}{u_{z,rms}'\rho_{rms}'}$
D9.6	40	1.6	9.6	4	1024×320	1.1	1.1	0.019	0.028	0.06
B3.0	12	1.5	2.97	5	512×128	1.2	0.9	0.025	0.026	0.08
A1.8	7.5	1.5	1.75	4	256×80	1.1	1.2	0.026	0.021	0.09
R0.7	2.8	1.6	0.69	2.5	128×64	1.1	1.1	0.039	0.011	0.13
R0.3	1.2	1.5	0.29	2	80×48	1.1	1.0	0.064	0.004	0.17

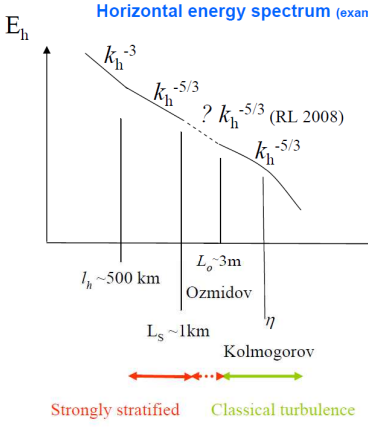
II- Turbulence en stratifié



$$E_K(k_h) = C_1 \varepsilon^{2/3} k_h^{-5/3}$$

$$E_K(k_v) \approx N^2 k_v^{-3}$$

III- Turbulence en stratifié tournant



$$L_o/l_h = F_h^{3/2} << 1$$

$$R_b = (L_o/\eta)^{4/3} >> 1$$

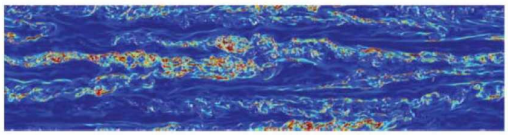


FIGURE 8. Snapshot of ε on an (x, z)-plane extracted from run D9.6. From low to high dissipation goes from dark blue, light blue to red.

III- Turbulence en stratifié tournant

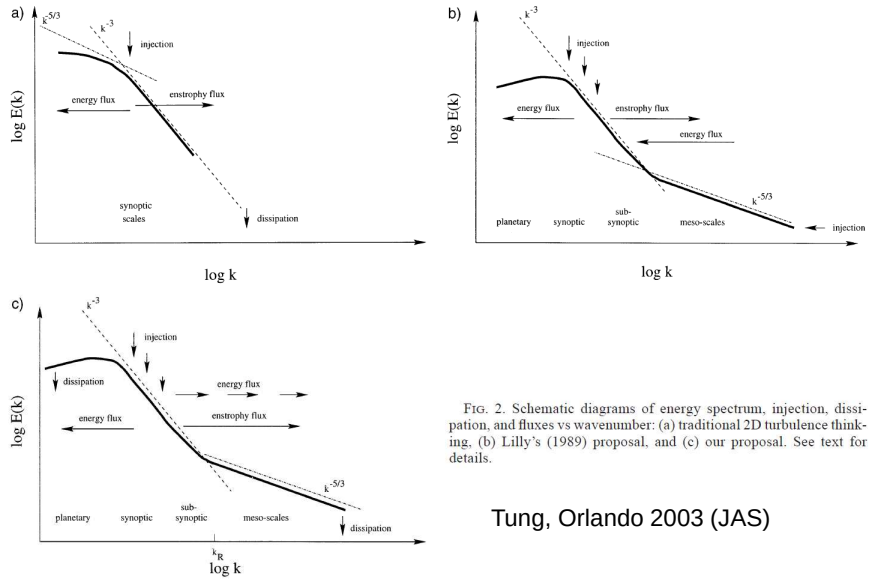


FIG. 2. Schematic diagrams of energy spectrum, injection, dissipation, and fluxes vs wavenumber: (a) traditional 2D turbulence thinking, (b) Lilly's (1989) proposal, and (c) our proposal. See text for details.

Tung, Orlando 2003 (JAS)

IV- et le mélange et le transport ?

Echelle de Thorpe L_T

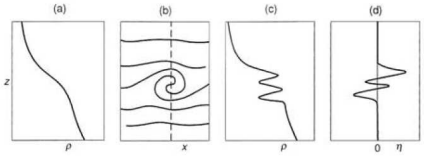


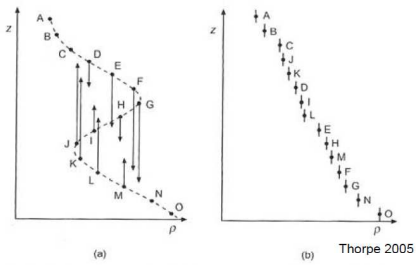
Figure 2.7. Displacements caused by an overturning eddy in stratified water. (a) The initial profile of density with density increasing with depth. (b) The distortion of isopycnals by the eddy and the path (dashed) of a free-fall instrument through the eddy. (c) The density profile measured from a sensor on the free-fall profiler. (d) The vertical displacements of density, η, resulting from the eddy. In practice, the initial density profile is unknown. Statically stable density profiles may be reconstructed from several observed profiles by reordering those measured into ones in which density increases with depth, supposing that no change in density has been brought about by molecular diffusion during the time over which the eddies develop, and then by finding the average depth of a set of selected values of the density in the several reconstructed profiles. (The latter procedure removes the smearing of wave-disturbed density interfaces that would be obtained by averaging densities at a set of fixed depths. Consider, for example, how to find the mean density profile across the wave-disturbed sea surface: averaging densities measured at fixed levels will smear the profile across a thickness equal to the wave height and lose the sharp change that, in reality, occurs at the sea surface.) The rms vertical displacements required in such a reordering provide measures of the vertical scale of eddies and of their potential energy. Thorpe 2007

$$\underbrace{2\delta_{i3}\left(g\frac{\overline{u_i'\theta'}}{\bar{\theta}}\right)}_{\text{turbulent buoyancy (heat) flux (source or loss)}}$$

Buoyancy production of TKE
- turbulent motions inc PE of fluid
- and/or PE loss of fluid raises energy of turbulent motions

PE of turbulent motion is given by $g\rho'\eta$

η is a vertical displacement of fluid particle with ρ



Thorpe 2005

Figure 6.2. The sorting algorithm. This is used to create a stable profile of density with $d\rho/dz \leq 0$ shown in (b) from the observed profile, (a), in which there is a statically unstable region where $d\rho/dz > 0$. The points, A–O, represent the discrete measured values of density at their respective levels, z. Those between C and N are statically unstable in the sense that, because of the density inversion, there is denser fluid above or less dense below them even though the density only decreases with depth between G and J. The vertical lines and arrows show the displacements in z required to re-sort the observed density profile into the statically stable order shown in (b). The vertical lines in (b) show the depth increments that are represented by the measured density values. These set the separations of the reordered points and need not necessarily be equal. They depend on the instruments' sampling interval in time, the instruments' response time, and the lowering rate. The latter may not be steady, for example when data is obtained by a lowered CTD in rough conditions. The sorting conserves mass but not the potential energy.

The sorting algorithm to determine L_T.

$$L_T = \left[\left\langle \eta^2 (\triangleq d^2) \right\rangle\right]^{1/2}$$

IV- et le mélange et le transport ?

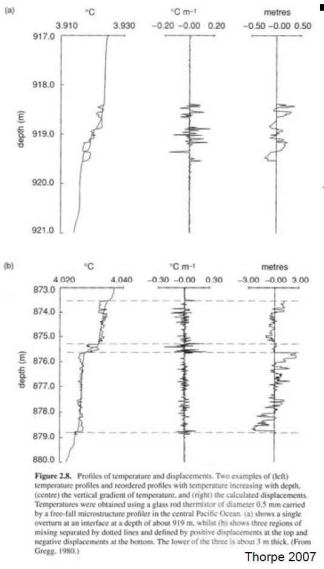


Figure 2.8. Profiles of temperature and displacements. Two examples of (left) temperature profiles and reordered profiles with temperature increasing with depth, (centre) the vertical gradient of temperature, and (right) the calculated displacements. Temperatures were obtained using a glass rod thermistor of diameter 0.5 mm carried by a free-fall microstructure profiler in the central Pacific Ocean. (a) shows a single overturn at an interface at a depth of about 919 m, whilst (b) shows three regions of mixing separated by dotted lines and defined by positive displacements at the top and negative displacements at the bottom. The lower of the three is about 3 m thick. (From Gregg, 1980.)

Thorpe 2007

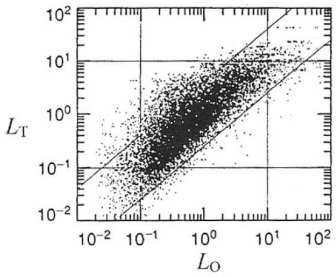


Figure 6.3. The variation of the displacement scale, L_T, with the Ozmidov scale, $L_O = \varepsilon^{1/2}N^{3/2}$, both measured in metres. (From Wesson and Gregg, 1994.)

Thorpe 2005

$$\varepsilon = c_1 L_T^2 N^3 = \left(\frac{L_o}{L_T}\right)^2 L_T^2 N^3 \approx 0.64 L_T^2 N^3$$

ie. $L_0 \approx L_T$

with scatter $0.25L_o < L_T < 4L_o$

IV- et le mélange et le transport ?

For density, $K_\rho = \dfrac{\Gamma\varepsilon}{N^2}$ (1) the so-called "diapyncal eddy diffusivity", when stable and Γ is an 'efficiency factor' (Osborn 1980)

TKE: $0 = B - \varepsilon + P + T$

In stable environment $B = \dfrac{g}{\rho_o}\overline{w'\rho'} < 0$

↳ TKE is suppressed!

With wind shear (2D), usually $P = -\overline{u'w'}\dfrac{\partial\overline{u}}{\partial z} > 0$

↳ TKE is produced!

Compare production/destruction terms due to buoyancy B and shear P:

$$R_f = -\frac{B}{P} = \frac{\dfrac{g}{\rho_o}\overline{w'\rho'}}{\overline{u'w'}\dfrac{\partial\overline{u}}{\partial z}} \qquad (2)$$

Static stability:

⇨ Statically stable (denom & num < 0): $R_f > 0$
Statically unstable (and always dynamically unstable): $R_f < 0$

Dynamic stability:

Richardson proposed R_f=+1 as critical value, as before, since ratio of TKE gain (which is MKE loss) balances buoyant consumption of TKE (B<0).

$0 < R_f < 1$ ⇒ B (<0) < P(>0)
⇒ TKE suppression < TKE production

$R_f > 1$ ⇒ B (<0) > P(>0)
⇒ TKE suppression > TKE production

↳ R_f < +1: Flow IS and REMAINS turbulent (dynamically unstable).
R_f > +1: Flow BECOMES laminar (dynamically stable).

With $K_\rho = \dfrac{-\overline{\rho'w'}}{d\overline{\rho}/dz} = \dfrac{g\overline{\rho'w'}}{\rho_o N^2}$

and (1) and (2) Thus, with $K_\rho = \dfrac{\Gamma\varepsilon}{N^2}$

$$K_\rho = \left(\frac{R_f}{1-R_f}\right)\frac{\varepsilon}{N^2} \qquad \Gamma = \frac{R_f}{1-R_f}$$

Measurements suggest Γ ≈ 0.2, but not all agree.
This value implies $R_f \approx 0.16$.