Wake of oblate spheroids and flat cylinders

M. Chrust, G. Bouchet and J. Dušek

Institut de Mécanique des Fluides et des Solides, Université de Strasbourg/CNRS, France

Parametric study of the transition scenario in the wake of oblate spheroids and flat cylinders
Summary

1 Introduction
   - Goal statement
   - Transition scenario of a sphere and a flat disk
Summary

1. Introduction
   - Goal statement
   - Transition scenario of a sphere and a flat disk

2. Numerical Method
   - Mathematical Formulation
   - Numerical Method
   - Computational domain
   - Boundary conditions
Summary

1. Introduction
   - Goal statement
   - Transition scenario of a sphere and a flat disk

2. Numerical Method
   - Mathematical Formulation
   - Numerical Method
   - Computational domain
   - Boundary conditions

3. Results
   - Oblate spheroids
   - Flat cylinders
Summary

1. Introduction
   - Goal statement
   - Transition scenario of a sphere and a flat disk

2. Numerical Method
   - Mathematical Formulation
   - Numerical Method
   - Computational domain
   - Boundary conditions

3. Results
   - Oblate spheroids
   - Flat cylinders

4. Conclusions
1 Introduction
   - Goal statement
   - Transition scenario of a sphere and a flat disk

2 Numerical Method

3 Results

4 Conclusions
Goal statement

Introduction

- **Study**: flow around oblate spheroids and flat cylinders.
- **Goal**: establishment of a link between transition scenarios of that of a sphere and a flat disk
- **Applications**: tool allowing to access the expected asymptotic state for any numerical or experimental configuration involving oblate spheroids or flat disks
Transition scenario of a sphere and a flat disk

**Sphere**

A 212 a 273 c 320 f 355 g Re

**Flat disk**

A 116.9 a 125.3 b 143.7 Meliga et al. Re

A 115 a 121 b 140 Fabre et al. Re
Transition scenario of a sphere and a flat disk

**Sphere**

**Steady axisymmetric (A)**

- **A**: 212
- 273
- 320
- 355
- **Re**

**Characteristics**

- steady
- axisymmetric
- the only non-zero mode is \( m=0 \)
- lift is equal to zero

Wake of oblate spheroids and flat cylinders

M. Chrust, G. Bouchet and J. Dušek
Transition scenario of a sphere and a flat disk

**Sphere**

**Steady non-axisymmetric (a)**

![Graph showing transition scenario parameters for sphere](image)

**Visualisation**

![Visualization of sphere wake](image)
Transition scenario of a sphere and a flat disk

**Sphere**

**Periodic state with planar symmetry (c)**

![Graph showing a periodic state with planar symmetry (c)]

**Visualisation**

![Visualization of the wake of an oblate spheroid and a flat cylinder]

(c), $\chi = 1.25$, $Re = 268$
Transition scenario of a sphere and a flat disk

Sphere

Quasi-periodic state (f)

\[ \chi = 1.25, \; Re = 268 \]

Visualisation
Transition scenario of a sphere and a flat disk

Sphere

Chaotic states (g)

Visualisation

\[(g), \chi = 1.5, Re = 310\]
Transition scenario of a sphere and a flat disk

**Flat disk**

**Periodic state without planar symmetry (b)**

\[ \chi = 6, \, Re = 145 \]

**Visualisation**

\[ C_\lambda^\infty \text{ versus } C_x \]

Wake of oblate spheroids and flat cylinders

M. Chrust, G. Bouchet and J. Dušek
Transition scenario of a sphere and a flat disk

Flat disk

Periodic state with a zero mean lift (d)

\[ \chi = 6, \ Re = 183 \]
Summary

1 Introduction

2 Numerical Method
   - Mathematical Formulation
   - Numerical Method
   - Computational domain
   - Boundary conditions

3 Results

4 Conclusions
Mathematical Formulation

- 3-d Navier Stokes equations are non-dimensionalized with respect to the inflow velocity $U$ and the diameter of the transverse cross section $d$
- solved in a cylindrical coordinate system $(z, r, \theta)$
- $Re = Ud/\nu$; $\chi = d/a$ where $d$ is the transverse diameter and $a$ the length of the streamwise axis of the spheroid

\[ \nabla \cdot \mathbf{v} = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \cdot \mathbf{v} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{v} \]  \hspace{1cm} (2)
the equations are discretized using the spectral-spectral-element discretization

Fourier expansion in the azimuthal direction

spectral-element discretization in the \((z, r)\)-plane by \(K\) standard high-order Lagrangian finite elements with \(N\) Gauss-Lobato-Legendre collocation points in each direction within element

time integration by semi-implicit method with a fully explicit third order accurate treatment of the non-linear coupling terms and with a full inversion of the Stokes problem in the individual azimuthal eigen-subspaces.
Computational domain

Discretization

**Figure:** Spectral element discretization of the computational domain of an ellipsoid of $\chi=2$.

- 215-241 spectral elements
- $L_{in} = 12d, L_{out} = 25d, R = 8d$
Mesh optimization

<table>
<thead>
<tr>
<th>Re</th>
<th>N=6</th>
<th>N=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re₁</td>
<td>117.17</td>
<td>116.92</td>
</tr>
<tr>
<td>Re₂</td>
<td>125.18 (4)</td>
<td>125.12 (4)</td>
</tr>
<tr>
<td></td>
<td>125.15 (6)</td>
<td>125.11 (6)</td>
</tr>
</tbody>
</table>

**Table:** Mesh test results for a flat disk. The values in brackets indicate the number of azimuthal modes of a Fourier expansion.

- Influence of the number of azimuthal modes m on the second instability threshold, m=4
- Number of collocation points, N=6
Boundary conditions

- inflow on the upstream of the cylindrical domain; uniform velocity profile \( u = (U_\infty, v = 0, w = 0) \)
- outflow on the lateral and downstream of the domain; pressure and stresses set to zero
- no-slip conditions on oblate spheroids/disks
- symmetry axis boundary condition using the complex velocity components of Orszag and Patera
Summary

1 Introduction

2 Numerical Method

3 Results
   - Oblate spheroids
   - Flat cylinders

4 Conclusions
FIGURE: State diagram for oblate spheroids. A stands for axisymmetric state. The filled triangles (c), and diamonds (e) represent pre-chaotic states with a subharmonic modulation. The narrow filled band represents the domain of bi-stability at the subcritical bifurcation.
**Figure:** Projection of the lift coefficient onto the perpendicular direction. $\chi = \infty$, $Re = 126$. 

Wake of oblate spheroids and flat cylinders

M. Chrust, G. Bouchet and J. Dušek
State with a non-zero helicity (e)

Characteristics

- unsteady
- unequal amplitudes of spiral modes (appearing in the weakly non-linear analysis)
- helicity yields an elliptic path of the lift
- slowly oscillating/rotating ellipse

Visualisation

\[ \chi = 1.25, \ Re = 283 \]
**Oblate spheroids**

**Bistability**

**Figure:** Oscillation amplitude $\Delta CL$ of the lift coefficient as a function of the Reynolds number for oblate spheroids of $\chi = \infty, 6, 3$ and a flat cylinder of $\chi = 6$ (see the legend). The stability interval of the steady non axisymmetric state is represented by dashed lines along the horizontal axis. Their linear instability thresholds are plotted as empty circles.
**Oblate spheroids**

**Subcritical bifurcation**

**Figure:** Logarithmic plot of the amplitude of the oscillation of the lift coefficient of a flat disk ($\chi = \infty$) at $Re = 126$ (full line) compared to the purely exponential growth (dashed line). Note the super-exponential growth in the time interval $t \in [1950, 2350]$ time units.
### Bistability thresholds

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$Re'_2$</th>
<th>$Re_2$</th>
<th>$\chi$</th>
<th>$Re'_2$</th>
<th>$Re_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>[124]</td>
<td>125.2</td>
<td>$\infty$</td>
<td>[124]</td>
<td>125.2</td>
</tr>
<tr>
<td>6</td>
<td>[136]</td>
<td>137.7</td>
<td>6</td>
<td>[148]</td>
<td>150.1</td>
</tr>
<tr>
<td>3</td>
<td>[154]</td>
<td>155.7</td>
<td>4</td>
<td>[164]</td>
<td>165.6</td>
</tr>
</tbody>
</table>

**Table:** Linear stability thresholds $Re_2$ and lower bounds of bi-stability $Re'_2$ ($Re'_2 < Re_2$) of the subcritical Hopf bifurcation for oblate spheroids and flat cylinders. (The values in brackets are closest integer upper bounds.)
### Results

**Comparison of the results with literature**

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$Re_1$</th>
<th>$Re_2$</th>
<th>$Re_3$</th>
<th>$Re_4$</th>
<th>$Re_5$</th>
<th>$Re_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$ (1)</td>
<td>$\approx 115$</td>
<td>$\approx 121$</td>
<td>$\approx 140$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$ (2)</td>
<td>116.9</td>
<td>125.3</td>
<td>143.7*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$ (3)</td>
<td>116.92</td>
<td>[124,125.2]</td>
<td>(142,143)</td>
<td>(165,170)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 (4)</td>
<td>135</td>
<td>155</td>
<td>172</td>
<td>280</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 (3)</td>
<td>129.6</td>
<td>[136,138.7]</td>
<td>154.4</td>
<td>188.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 (5)</td>
<td>$\approx 159.4$</td>
<td>$\approx 179.8$</td>
<td>(184,185)</td>
<td>(190,191)</td>
<td>$\approx 215$</td>
<td>$\approx 240$</td>
</tr>
<tr>
<td>3 (3)</td>
<td>159.65</td>
<td>(181,182)</td>
<td>(185,190)</td>
<td>(195,198)</td>
<td>(220,230)</td>
<td>(235,240)</td>
</tr>
</tbody>
</table>

**Table:** Bifurcation thresholds. Numbers in brackets indicate authors: (1) Fabre et al., (2) Meliga et al., (3) present study, (4) Shenoy and Kleinstreuer, (5) Auguste et al. *result obtained using asymptotic expansion. At $\chi = 10$, the values of the present study are obtained by interpolation between $\chi = \infty$ and $\chi = 6$. 
Flat cylinders

Flat cylinders - bibliographic data

**Figure**: State diagram for cylinders. Bibliographic data.
Figure: State diagram for cylinders. The band of bi-stability at the subcritical bifurcation is still present for $1/\chi \leq 0.25$. 

Wake of oblate spheroids and flat cylinders
M. Chrust, G. Bouchet and J. Dušek
Summary

1. Introduction
2. Numerical Method
3. Results
4. Conclusions

Wake of oblate spheroids and flat cylinders
M. Chrust, G. Bouchet and J. Dušek
1. The parameter space \((\chi, Re)\) has been explored for oblate spheroids and flat disks.
2. The secondary bifurcation is subcritical for \(1/\chi \leq 0.45\) and \(1/\chi \leq 0.25\) for oblate spheroids and flat disks respectively.
3. The scenario of a flat disk and sphere are separated by the existence of the state with non-zero helicity.
Future work

- Investigation of the transition scenario of freely falling disks and spheroids
Merci