Institut de Mécanique des Fluides - Amphithéâtre Nougaro
Allée du Pr. Camille Soula, Toulouse

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Beyond effective diffusivity — Dispersion in the large-deviation regime.

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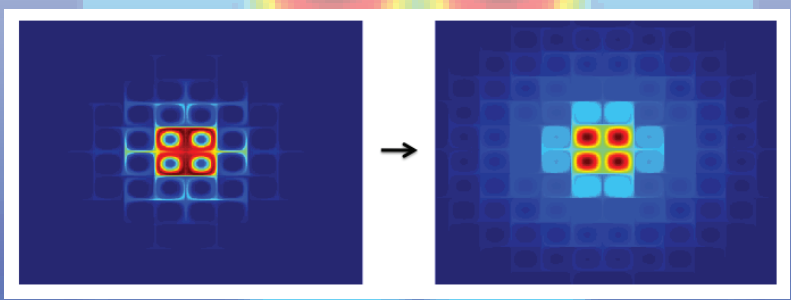
The dispersion of a passive scalar in a fluid through the combined action of advection and molecular diffusion is often described as a diffusive process, with an effective diffusivity that is enhanced compared with the molecular value. However, this description fails to capture the tails of the scalar concentration distribution in initial-value problems. These tails would be important, for example, if what was of interest was the first time at which a small concentration threshold was exceeded. They are also important in problems with reaction.

To extend the diffusive description we develop a large-deviation theory of scalar dispersion that provides an approximation to the scalar concentration valid at much larger distances away from the centre of mass, specifically distances that are O (t) rather than O ($t^{1/2}$) where t is the time from the scalar release. The theory centres on the calculation of a rate function characterizing the large-time form of the scalar concentration. This function is deduced from the solution of a one-parameter family of eigenvalue problems. We emphasize the connection between the large-deviation theory and the homogenization theory that is often used to compute effective diffusivities: a perturbative solution of the eigenvalue problems in the appropriate limit reduces at leading order to the cell problem of homogenization theory. We consider two classes of flows in some detail: shear flows and periodic flows with closed streamlines (cellular flows). In both cases, large deviation generalizes classical results on effective diffusivity and captures new phenomena relevant to the tails of the scalar distribution. These include approximately finite dispersion speeds arising at large Péclet number Pe (corresponding to small molecular diffusivity) and for two-dimensional cellular flows, anisotropic dispersion. Explicit asymptotic results are obtained for both shear flows and cellular flows in the limit of large Pe.

(Work in collaboration with Jacques Vanneste, University of Edinburgh.)



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