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WEAKLY NON-LINEAR ANALYSIS OF THE FLUTTER MOTION OF DISKS

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Introduction

The problem of a freely falling or rising body in a viscous fluid is quite fascinating. Even simple objects like disks (or more exacly flattened cylinders) falling in a newtonian fluid have been known for centuries as able to undergo chaotic trajectories and regular ones such as flutter, tumbling and oblique drift. From falling leaves and seeds in ecology to the freefall of a spacecraft through rarified atmosphere in aeronautics, the wide range of applications of this problem has led, particularly in the last decade, to several experimental, numerical and theoretical works (see Ern *et al.* 2012, for a comprehensive review). Tackling this complex problem from a stability point of view, we recently carried out the linear stability analysis of the coupled fluid-body problem in the case of two-dimensional objects such as thin plates and rectangular rods (Assemat *et al.* 2012) and axisymmetric bodies, namely disks of finite thickness (Tchoufag *et al.* in preparation). Here, we extend our study to the primary nonlinear regimes, whose amplitudes are assumed to result from the saturation of the linear modes and computed by means of a weakly non-linear analysis.

1 Full text

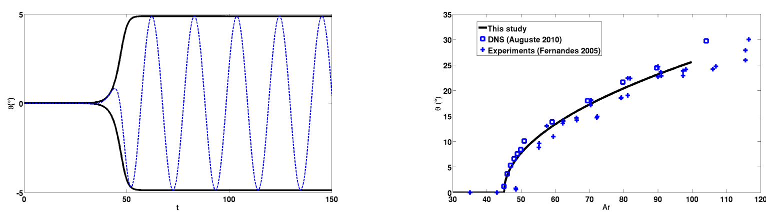
The analysis of the coupled fluid-disk system is performed for an incompressible and newtonian fluid, of viscosity ν and density ρ_f. The elasticity of the considered body being neglected (no possible buckling and flapping), the trajectory of the disk is thus governed by the rigid-body motion equations and the flow around it by the Navier-Stokes equations. The system is fully coupled through the fluid forces and torques one hand and and the boundary conditions imposed by the disk on the other hand. Let D and h be the disk diameter and thickness. m_{disk} its mass, and g the gravity acceleration. It can be shown through dimensional analysis that the trajectory is sufficiently described by three dimensionless numbers: aspect ratio, dimensionless moment of inertia and "Archimedes" number (or gravitational Reynolds).

$$\chi = \frac{D}{h} \qquad I^* = \frac{m_{disk}}{16\rho_f D^3} \qquad Ar = \left(\frac{3}{16}\right)^{\frac{1}{2}} \frac{D}{\nu}\sqrt{\frac{\Delta\rho g D}{\rho_f \chi}}$$

The talk will expose the formalism of the weakly non-linear analysis (hereafter referred to as WNLA) which is in the line of the work by Sipp and Lebedev (2007) who dealt with the well-known case of the Von Karman instability in the wake of a 2D cylinder. However, since our equations are written for 3D and freely moving objects, they generalize those of the latter authors. In brief, the WNLA consists in solving in an incremental manner, the flow which is searched as a third order expansion in increasing powers of a "small" control parameter. Let $\mathbf{Q} = (\mathbf{V}(\mathbf{r},t), P(\mathbf{r},t), \mathbf{U}(t), \mathbf{\Omega}(t), \mathbf{\Phi}(t))^T$ be the state vector made of the velocity and pressure in the fluid, the translational and rotational velocities of the object and its inclination with respect to the vertical axis.

$$\mathbf{Q} = \mathbf{Q}_0 + \epsilon \mathbf{Q}_1 + \epsilon^2 \mathbf{Q}_2 + \epsilon^3 \mathbf{Q}_3 + ...$$

For the sake of conciseness, let us just say that, using a finite elements software (FreeFem++) and a Krylov-Schur eigensolver (SLEPc), we compute the base flow (which corresponds to the steady vertical freefall) at order 0, the linear global modes (normalized for a unit inclination) at order 1 and harmonics as well as some other flow corrections at order 2. In our problem, we choose $\epsilon \simeq \delta / Ar_c^2$ where Ar_c is the threshold for the base flow to be unstable. It thus goes without saying that this WNLA was preceeded by a linear analysis where the thresholds are determined for various χ and various I^*. Finally, the amplitude equation, is derived at order 3 through the projection on the adjoint global mode of the resonant forcing. Due to our deliberate normalization, the amplitude equation then gives the temporal evolution of the inclination ϕ of the disk on its trajectory. Note that, Fabre *et al.* 2012 also predicted the inclination of the disk which undergoes a steady oblique motion, through a different formulation of the WNLA. The figures below show $\phi(t)$ as the disk goes from a vertical ($\phi = 0$) to a converged flutter state and a successful comparison between our asymptotic prediction, DNS results(Auguste 2010, Chrust 2012) and experiments (Fernandes 2005).



(a) (b)

Figure 1: Thick disk $\chi = 3$ and $I^* = 0.0186$ (a) Temporal evolution of the amplitude of inclination at $Ar = 47$. (b) Increase of the inclination with Ar. The transition from a straight path to a periodic flutter is $Ar_c \simeq 45$.

2 Conclusion

In the presentation, we shall focus of prediction of the flutter motion (amplitude and Strouhal number) also known as the Zig-Zag regime. It will be shown that our third order WNLA is able to predict the inclination of fluttering thick disks ($\chi \in [2, 6]$) and less reliable when it comes to thin disks ($\chi \geq 10$). Explanations to this failure and possible ways of improvement will also be given.

References

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