Instability of a potential swirling flow with a free surface

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Rotating flows with free surface are commonly observed in everyday-life, the most popular example being the drain vortex observed in bathtubs and sinks. These flows are known to support spectacular patterns arising from breaking of the axial symmetry. A situation which has been particularly investigated through experiments is the case of a partially filled container with a rotating bottom. In this situation, an instability leads to the appearance of rotating polygons along the free surface [1] [2]. The axisymmetric base flow preexisting the instability is rather complex, involving strong recirculations, but includes a region where the rotation is approximately potential, [3] suggesting that the potential vortex may be a relevant model to investigate the instability mechanism.

Consequently, we investigate here the linear inviscid stability of the potential swirling flow with a free surface; namely, the flow defined $V_\theta = \Gamma/(2\pi r)$, with a free surface given by $h_0(r) \sim \frac{1}{R_1^2} - \frac{1}{r^2}$, where R_1 corresponds to the radius of the dry area. For the study, we use both a global stability approach based on the finite element software Freefem++ and a shallow water approach. Our results show that unstable modes exist in narrow ranges of the Froude number (defined by $Fr = \Gamma/2\pi\sqrt{(gR_0^3)}$, R_0 being the radius of the cylinder), for all non-zero values of the azimuthal wavenumber m (see figure 1). The instability seems to be related to the "strato-rotational" instability mechanism existing in stratified rotating fluids[4].

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[1] Vatistas et al, *PRL* **100**, 174503 (2008).

[2] Jansson et al, *PRL* **96**, 174502 (2006).

[3] Tophøj, *Phd Thesis*, Danmark Technical University (2012).

[4] Le Dizés and Riedinger, *J. Fluid Mech.* **660**, 147 (2010).

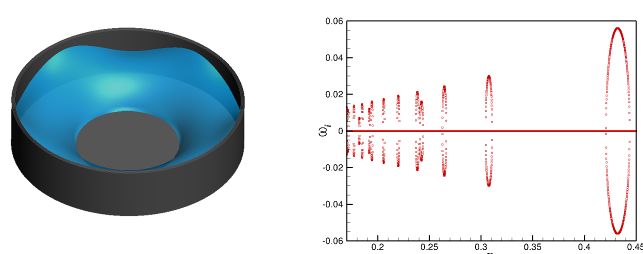

Figure 1: (a) 3D free surface reconstruction of the most amplified mode for an azimuthal wave number $m = 5$ (b) Amplification rate as a fonction of the Froude number for $m = 5$ (the liquid volume in the container is kept constant as the Froude number is varied).