AIAA 2001-1038

The Properties of a Transport Aircraft Wake in the Extended Near Field: an Experimental Study

L. Jacquin, D. Fabre, P. Geffroy
ONERA/DAFE – 8 rue des Vertugadins, 92190 Meudon, France

E. Coustols
ONERA/DMAE – 2 avenue Edouard Belin, 31055 Toulouse, France.
SUMMARY

An experimental study on the formation and development of the vortex wake behind a small scale transport aircraft model is presented. The data comprises LDV and hot wire measurements including two-point correlations. Steady and unsteady properties of the wake have been characterised for two cases, a high lift and a clean configuration of the model. The analysis particularly focuses on the unsteady properties of the vortices which are subjected to meandering. The discussion addresses elements that contribute to a better understanding of this phenomenon.


1. INTRODUCTION

The optimisation of airport traffic motivates research on vortex wakes with the goal to reduce the current spacing between airplanes in complete safety. Aeronautics manufacturers’ concerns are to ensure that future certification regulations will not penalise their airplanes due to their “vortex signature”. Large transport aircraft projects are the object of particular concern on this matter. Industry is therefore interested in any development that might help the control of the wakes emitted by transport aircraft. In view of the major industrial interests, the knowledge of physical laws which govern the formation, the development and the dispersion of wakes is still too limited. Some helpful reviews on this matter include Olsen et al. (1971), Donaldson and Bilanin (1975), Widnall (1975) and Spalart (1998). This paper aims at presenting a compilation of experimental results about the vortices generated by a small scale representative transport aircraft model for both landing and cruise configurations. The experimental performance comprises LDV and hot wire. The results concern the statistical properties of the wake for the mean field and the turbulent quantities. It focuses on the meandering of the vortices, which is the main manifestation of unsteadiness of wake vortices observed in wind tunnel experiments. This mechanism, which could participate to the vortex wake disorganisation, is not fully understood.

2. VORTEX WAKE HAZARD

In this section, we consider a typical situation where a "following aircraft" encounters the vortex wake of an "emitting aircraft", as represented in figure 1. We identify the parameters which characterise the potential hazard of this encounter.

2.1. Rolling moment

As represented in figure 1, the far wake of the “emitting” aircraft is modelled by a pair of counter-rotating vortices separated by a distance $\tilde{b}$. A following aircraft, with a wing span $l$ assumed to be small in comparison to $\tilde{b}$ and aligned on the axis of one of the vortices, is affected by a rolling effort characterised by the following coefficient (Condit & Tracy, 1971):
where \( V \) designates the velocity of the following aircraft, \( S \), the wing area and \( c(y) \), its local chord. Considering a rectangular wing \( (c(y) = Cte) \) and approximating the velocity field with two point vortices of circulation \( \pm \Gamma \), gives \( C_l = \Gamma / \mathcal{V} \). This relation shows that, given the span of the following aircraft, the « hazard» is proportional to the circulation \( \Gamma \) of the incident wake.

### 2.2. Lengthscales

Considering vortex points, as above, amounts to make the assumption that the characteristic length scales of the vortex cores are negligible compared to the span of the following aircraft. This may not be the case and the estimation of the hazard has to be refined in consequence. Several lengthscales may be defined in order to characterise the flow. The first length scale characterising the vorticity field is the separation \( \bar{b} \) between the vorticity centroids (see e.g. Saffman, 1992):

\[
\bar{b} = \frac{1}{\Gamma} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \omega(y) \, dy \, dz
\]

in which \( \Gamma \) is the circulation in a half plane, see figure 1. Considering an elliptic loading of the emitting aircraft with span \( b \) gives \( \bar{b} = (\pi / 4) \bar{b} \). For an airplane with velocity \( V_\infty \), lift coefficient \( C_L \), airfoil surface \( S \) and aspect ratio \( AR = b^2 / S \), the lift, \( 1/2 C_L \rho V_\infty^2 S \), is equal to the flux of the wake vertical momentum \( \rho V_\infty \Gamma \bar{b} \). It follows that:

\[
C_l = \frac{4\pi}{S\mathcal{V}} \int_{0}^{\pi/2} y \, c(y) \nu(y,0) \, dy
\]
In the case of an elliptic loading, the circulation is therefore given by:

\[ \Gamma = \frac{\text{Lift}}{\rho V_\infty b} \]  

(3)

For a landing transport aircraft one typically has: \( C_L \approx 2, \quad AR = 7 \).

Other scales are characteristic length scales of the vortex cores. Several vortex core definitions can be proposed. An aircraft vortex is generally assumed to consist of an 'internal core' or 'viscous core' of radius \( r_1 \), rotating as a solid body, and of an 'inviscid core' or 'external core' of radius \( r_2 \), the region containing vorticity surrounding the 'internal core' (see Spalart, 1998). The internal core radius \( r_1 \) is usually defined as the position of maximum azimuthal velocity, and the external core radius \( r_2 \) as the position where the total circulation of the vortex is almost attained. These two characteristic length scales can differ substantially. This will be shown in the course of the presentation of our experimental results. The physical meaning of the \( r_2 \) scale is not obvious, but it reflects the existence of vorticity « far » from the vortex axis. Other definitions, which reflect this fact in a more precise way, have been proposed. The most widely used is the 'dispersion radius', defined as:

\[
\tilde{r}^2 = \frac{1}{\Gamma} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( y - y_c \right)^2 + \left( z - z_c \right)^2 \omega \, dy \, dz
\]  

(5)

This scale provides an evaluation of the vorticity field dispersion in the plane. Here \( y_c \) and \( z_c \) are the coordinates of the vorticity centroid in half a plane. This radius is interesting from a theoretical point of view because one can show that, if the evolution of the wake is purely bidimensional, it can only increase by viscous diffusion (see Saffman, 1992).

Other definitions have been proposed. Widnall et al. (1971) and Moore & Saffman (1972) introduced the 'effective core radius':

\[ r_{\text{eff}} = r_2 \exp\left\{1/4 - A + C\right\} \]  

(6)

with \( A = \left( \frac{2\pi}{\Gamma} \right)^2 \int_0^{r_2} V_\theta^2 (r) r \, dr, \quad C = 2\left( \frac{2\pi}{\Gamma} \right)^2 \int_0^{r_2} U^2 (r) r \, dr \). Here \( V_\theta \) is the azimuthal velocity component and \( U \) is the axial velocity deficit. Both are assumed to be axisymmetric. This 'effective radius' has two interesting properties. First, it is related to long-wave stability properties (see §6.3). Secondly, if the wake is free from axial flow (\( C = 0 \)) and consists of a single pair of trailing vortices, this radius is directly related to the flux of kinetic energy in the wake (see Widnall et al., 1971). It can then be determined by balancing the energy flux with the
induced drag of the aircraft. Considering a wing with an elliptic loading, this leads to an estimation of the effective core radius \( r_{\text{eff}} = 0.11 \bar{b} \), a value retained by most authors as the 'vortex core radius' \(^1\).

A last definition for the vortex core size is proposed here. We call the 'rolling moment radius' a quantity defined as :

\[
r_{\text{roll}} = \frac{1}{\Gamma} \int_{-\infty}^{\infty} \int_{0}^{\infty} \left( \frac{\omega}{\Gamma} \right) \left( \sqrt{(y-y_c)^2 + (z-z_c)^2} \right) \, dy \, dz,
\]

This quantity is of particular interest if one is interested in evaluating the hazard that a vortex wake represents for a following aircraft. Going back to Equation (1), and considering now that the span \( l \) is comparable to the vortex core scales but larger than the external core \( (l \geq 2r_2) \), one obtains :

\[
C_l = \frac{1}{\Gamma} \left[ l - 2r_{\text{roll}} / l \right]
\]

Thus the 'rolling moment radius' appears as a correction in the expression of the rolling moment compared to the case of point vortices. The larger is \( r_{\text{roll}} \), the smaller is the rolling moment. Consequently, in order to reduce hazards, we either need to reduce the overall circulation of vortices, or increase their dispersion, as quantified by the 'rolling moment radius' proposed here. For example, for the two scale model proposed bellow, see relations (10) of §4.2, it may be shown that the 'rolling moment radius' defined in (7) reads :

\[
r_{\text{roll}} = r_2 / 3 \left[ 1 + (r_1 / r_2)^{3/2} \right]
\]

If the internal core size is small, which turns out to be the case as shown by our experiences where \( r_1 / r_2 = O(10^{-1}) \), see §5, \( r_{\text{roll}} = r_2 / 3 \). If one considers a following aircraft of the size of the region containing vorticity, i.e. \( l = 2r_2 \), the rolling coefficient given by (8) is decreased by one third with respect to the point vortex value \( C_l = 1/\Gamma \).

In summary, five different core radii may be considered : \( r_1, r_2, \bar{r}, r_{\text{eff}}, \) and \( r_{\text{roll}} \). Note that the available experimental and numerical results show a discrepancy on the internal core radius \( r_1 \). On the other hand, the four other scales are generally found to be of the order \( O(0.1\bar{b}) \).

### 2.3. Reduction of circulation

Considering relation (3), the circulation in the wake behind an aircraft in steady conditions\(^2\) is inversely proportional to the wing span. This speaks in favour of large wing spans.

---

\(^1\) This energy balance argument is due to Prandtl, and correctly reproduced by Donaldson & Bilanin (1971). As noted by Spalart (1998), the paper from Spreiter & Sacks (1951), often referenced on this point, contains a slight computational mistake.
Once $b$ has been fixed, when integrating the convection-diffusion vorticity equation in the half plane in the case of a localized two-dimensional flow field that is symmetrical with respect to the plane $y = 0$ (see Fig.1), the only remaining mechanism is diffusion by viscosity through the plane of symmetry: $\frac{\partial \Gamma}{\partial t} = \nu \int_\infty^\infty \frac{\partial \omega}{\partial y} \, dz$ (see e.g. Donaldson & Bilanin, 1975). As a result, given $b$, $\Gamma$ can only decrease through diffusion and cancellation of circulation through the plane that separates the two vortices. In practice, the Reynolds number is very high (typically $Re = \frac{\Gamma}{\nu} = 10^7$ to $10^8$) and viscous diffusion is a very slow process. Unfortunately, as discussed briefly in §6.2, there is nothing to expect from the development of the turbulence within the vortex. But one may think of promoting turbulence in the very first stage of the formation of vortices through the addition of « winglets » or by bringing the axis of the aircraft engines and the wing tip closer together. However, the persistence in the far field of a positive effect that could be obtained by this way remains to confirm.

2.4. Linear instabilities

Another strategy is the promotion of linear instabilities. Three-dimensional linear instabilities can speed up the transfer process across the center plane through a change in the vortices’ topology. As discussed below, these instabilities develop on the $\tau_b \propto \frac{b^2}{\Gamma}$ time scale (see §6.3). In comparison with the viscous time scale, e.g. $\tau_v \propto \frac{1}{\nu}$ with $r$ the dispersion radius (5), we have $\tau_v / \tau_b \propto \left(\frac{r}{b}\right)^2 Re$. Therefore, if we assume that $\frac{r}{b} = O(0.1)$, for $Re >> 100$, instabilities are a potentially more efficient dispersion mechanism than diffusion.

We then see that this leads to a recommendation that conflicts with the previous one regarding wing span. Minimizing the characteristic time of instabilities $\tau_b \propto \frac{b^2}{\Gamma}$ speaks in favour of a reduction in wing span: for a given weight, according to (3), $\Gamma \propto b^{-1}$, therefore $\tau_b \propto b^3$!

Instabilities may be initialised by atmospheric turbulence, but we can use moving devices (Bilanin & Widnall, 1973) and/or special arrangement of the vortices (see Rennich & Lele, 1998, Fabre & Jacquin, 2000) to speed up their development. However, we must indicate that the turbulent cascade process needed for the final dispersion of the wake once the linear instabilities have developed has not yet been fully identified (see Orlandi et al., 1998).

3. PRESENTATION OF THE EXPERIMENT

3.1 Wind tunnel and model

If $c$ is the aerodynamic chord length and $b$ the airplane’s wing span, one usually distinguishes four areas in a vortex wake: (i) the near-wake field, which pertains to distances where $x/c = O(1)$ and is characterised by the formation of highly concentrated vortices downstream of all surface discontinuities; (ii) the extended near-wake field, covering distances where $x/b \leq 10$, in which the roll-up of the trailing system takes place and where the merging of the dominant vortices (wings, flaps) occurs leading gradually to two counter-rotating vortices;
the far wake-field, where $10 \leq x/b \leq 100$, in which the vortex pair descend in the atmosphere without undergoing any major change and where linear instabilities emerge; (iv) the dispersion regime, $x/b > 100$, where fully developed instabilities result in a strong interaction between the two vortices until they ultimately collapse.

The experiment concerns the formation and evolution of a wake in the extended near field as defined above. It was conducted in the F2 wind tunnel at ONERA – Le Fauga-Mauzac centre. This wind tunnel has a cross section of $1.8m \times 1.4m$ for a length of $5m$. The model, provided by EADS Airbus S.A. Centre de Toulouse, is a small scale generic model (1:100) of an Airbus A300 (wing span $b=448$ mm, aerodynamic chord $c = 66$ mm). Such model dimensions allow to investigate the flow 9 wing spans downstream of the model. Two configurations were examined: a cruise configuration (clean case: angle of attack $\alpha=4^\circ$, lift coefficient $C_L \approx 0.7$ ) and a high lift configuration with wings fitted with single block flaps (high lift case : $\alpha=8^\circ$, $C_L \approx 1.7$ ). The model has no horizontal tail but comprises two permeable engine nacelles. Tests have been performed for a free-stream velocity $V_\infty = 50ms^{-1}$, the Reynolds number based on the aerodynamic chord being $Re_c \approx 200000$. The transition of the boundary layer has been tripped on the fuselage nose, wing, vertical fin and nacelles. Oil-flow visualizations have been used to check transition tripping and to observe the flow quality on the model wings and flaps. Figure 2 shows the model in the working section.

3.2. Measurement techniques

The experimental programme included smoke visualizations, Laser Doppler velocimetry (LDV) and hot wire measurements in the wake of the model in clean and in high lift configurations. This test program is summarized in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Clean</th>
<th>High Lift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model incidence</td>
<td>$4^\circ$</td>
<td>$8^\circ$</td>
</tr>
<tr>
<td>Lift coefficient</td>
<td>$\approx 0.7$</td>
<td>$\approx 1.7$</td>
</tr>
<tr>
<td>Station $x/b$</td>
<td>0.5</td>
<td>1 3 5 9</td>
</tr>
<tr>
<td>Smoke visualisations</td>
<td>× × × ×</td>
<td>× × × ×</td>
</tr>
<tr>
<td>LDV</td>
<td>×</td>
<td>× × × × ×</td>
</tr>
<tr>
<td>Hot Wire</td>
<td></td>
<td>×</td>
</tr>
</tbody>
</table>

*Table 1 : Wind tunnel test program*

3.2.1. Smoke visualisations

Visualizations based on the laser sheet technique have been conducted to characterize the vortex wake unsteadiness. The tunnel was filled up with incense smoke so that the vortex central core, with a deficit of particles due to centrifugal effects, could be visualized and recorded. The video has shown that the vortices are subject to a meandering which increases with downstream distance.
3.2.2. Laser Doppler Velocimetry

The F2 wind tunnel is equipped with a traversing 3-component LDV system which covers nearly the whole test section. The LDV arrangement is schematised in figure 3. It uses two argon lasers, one operating all lines which delivers 3W on the green (514.5nm) and blue (488nm) lines, and the second one, operates at its maximum power on the violet line (476.5nm) with a power 2.7 W. The system works in the forward-scatter mode. As figure 3 shows, depending on the measurement volume location, the 2D component beam plane can be oriented either horizontally, figure 3(b), or vertically, see figure 3(c). The fringe triangled has angles which vary between 30° and 60°. An error usually occurs when the velocity component is not measured directly, that is when the fringe triangled is not orthogonal, as it is the case here. The arrangement described in figure 3 may lead to a 30-50% error in the determination of $\sqrt{\nu^2}$. The sources of this error are still ill-defined (measurement noises, «virtual particles») and is the subject of undergoing studies in the LDV community. Other errors specific to the measurements of highly concentrated vortices, e.g. particle centrifugation and spatial integration, are discussed in the following.

The high lift and clean configurations were surveyed in 5 planes normal to the free-stream direction $x$, 0.5, 1, 3, 5 and 9 wing spans downstream of the model. The origin of the reference frame is the intersection of the line connecting both downstream ends of the wing tips and the plane of symmetry $y = 0$. Different mesh sizes have been used. The larger grids $\Delta y = \Delta z = 10\text{mm}$ have allowed to locate flow regions that had to be detailed. Smaller meshes, $5\text{mm}$ and $2.5\text{mm}$, were then used. The finest mesh, $\Delta y = \Delta z = 2.5\text{mm}$, was applied to the regions containing the wing tip and flap tip vortices.

Space-time correlations have also been measured in order to obtain more information on the vortex meandering. A special LDV arrangement has been used for this purpose. This arrangement schematised in figure 4 is based on the use of two independent optical systems which measure one component of the velocity at two points along the vortex trajectory with variable separations. One measurement volume, provided by a 1D fringe laser set-up is set fixed at given distance behind the model. The second measurement volume is provided by the wind tunnel LDV system described above, but with only one colour operating; it is moved along the vortex trajectory. The two systems were set to measure only the vertical component of velocity, $W(x,t)$, which is very sensitive to vortex meandering. The two LDV signals are processed simultaneously by a TSI-IFA 750. The results presented in §7 have been obtained with the fixed measurement volume fixed in the starboard vortex centre $x/b = 5$ downstream of the model, the second measurement volume being translated upstream up to the wing.

Incense smoke has been used for the seeding. One point statistical quantities have been calculated using 2000 samples while the space-time correlations have required acquisitions of $10^6$ samples detected simultaneously by the two 1D fringe laser systems for each separation distance between the two measurement volumes.
3.2.3. Hot wire

The vortex regions at $x/b = 1$ and 5 have also been investigated by means of a 55M DISA anemometer and a standard single wire DISA P11 probe. The probe is moved by means of a three directional traversing system which is mounted in place of a lateral side glass window of the test section.

Some tests have been made to check if the vortices were influenced by intrusive hot wire probe. This has been done by means of simultaneous measurements with the LDV system and a hot wire. Thus, the LDV configuration has been modified in order to approach the LDV measurement volume in the vicinity of the hot wire. During these tests, the probe moved together with the LDV measurement volume. Slight differences were found comparing the results with and without the probe, but they were small. In particular, the meandering of the vortex, which is characterized by a high level of the fluctuation kinetic energy (see below) was not significantly modified by the presence of the probe (see Jacquin et al., 1998). Devenport et al. (1996) made similar tests and they also concluded that probe/vortex interaction is weak.

4. MEAN FIELD AND FLUCTUATIONS

4.1. The mean vorticity field

Figure 5 shows the distribution of the axial component of normalised vorticity $(\Omega_\perp b/2)/V_\infty$ in the high lift case in the five downstream half vertical planes. As shown by figure 5(a), vorticity in the near field comprises various contributions but is dominated by the co-rotating vortices generated by the wing and flap tips. These vortices are of similar intensity: as indicated in that figure, the normalised vorticity in the wing tip vortex is worth -56, against -37 for the flap. These two vortices merge between $x/b = 1$ and 3 (tests have shown that the merging is completed just before $x/b = 2$). A maximum value of vorticity of -45 is obtained at $x/b=3$ (Fig.5(c)). The intensity of the resulting vortex then remains constant. This is illustrated on figure 6 which shows the constancy of the circulation $(\Gamma)$ calculated by the integration of velocity on circular contours. Note that, although the circulation remains unchanged from $x/b = 3$, figure 5(e) shows that at $x/b = 9$ the peak vorticity is equal to -20, compared to -45 at $x/b = 3$ and 5, see figures 5(c) and (d). This is due to a larger mesh size used at this position (5 mm against 2.5 mm in the other planes). The mesh size has a direct impact on the result of the velocity differentiation which provides vorticity. We can also notice the persistence, up to $x/b=9$, of a low intensity co-rotating vortex. This vortex comes from the through flow nacelle (TFN) of the model.

The circulation may be evaluated using equation (4). For the model in high lift configuration, $C_L=1.7$, $AR=7$, $V_\infty=50 ms^{-1}$, $b=0.448 m$. This leads to $|\Gamma| = 3.46 m^2 s^{-1}$, i.e. $|\Gamma|/(V_\infty b) = 0.15$. We can see from figure 6 that this value is not reached at $r/b = 0.3$. Note that inspection of figures 4(c)-(e) gives approximately $\delta = 340 mm$ for the vortex centre separation; this corresponds to $\delta/b = 0.76$, a value close to the theoretical one for the elliptic case, $\delta/b = \pi/4$. 
Now, considering the vorticity dispersion radius \( r \), applying equation (5) to the results presented in figure 5 leads to \( r/b = 0.1 \) for \( x/b \geq 3 \).

### 4.2. The mean flow structure of the final vortex

Figure 7 shows various results on the tangential velocity \( V_\theta \). Figures (a) and (b) show scatter plots of \( V_\theta \) at \( x/b = 5 \) for the two configurations. The tangential velocity at each measured grid point is plotted with respect to the distance to the vortex centre. As we come closer to the plane of symmetry (large values of \( r/b \)), the dispersion increases due both to the effect of the field in the fuselage region and to the influence of the symmetrical vortex. On the other hand, for small \( r \), the vortex is more or less axisymmetric and the dispersion is weaker, particularly for the clean case. Figure 7(c) and (d) show the result of a cylindrical averaging of the scatter plots obtained in the three last sections (\( x/b = 3, 5, 9 \)). Figure (e) and (f) show log-log plots of these last curves. The high lift case is characterised by the presence of a plateau with a width of \( r/b = 0.03 \) where \( V_\theta \) remains around its maximum value, that is approximately 23\% of \( V_\infty \), see figure 7(c) and (e). This plateau is not present in the clean case. Such a velocity plateau was also observed by Devenport et al. (1996); as suggested by these authors, this could correspond to a remnant of the initial conditions due to a merging of several cores.

After this plateau region, the decrease of \( V_\theta \) seems to follow a power law \( r^{-\alpha} \) with \( \alpha \) around 0.5. Exact values of the slopes are indicated in the figures 7(e) and (f). Departure from this law towards the potential law \( r^{-1} \) occurs around \( r/b = 0.1 \) (or \( r = 45 \text{mm} \)). Note on figure 7(c)-(e) that the radius where \( V_\theta \) decreases towards 0 when \( r \to 0 \), seems to be greater at \( x/b = 9 \). This is simply due to the different probing mesh used at this location. In fact, for the three curves, the radius where \( V_\theta \) begins to decrease is of the order of the probing mesh size: 2.5 mm at \( x/b = 3 \) and 5, and 5 mm at \( x/b = 9 \). This is an indication that the internal core radius of the vortex is very small.

In the clean case, a \( r^{-\alpha} \), with \( \alpha \) slightly larger than 0.5, is particularly well adapted here in the absence of a plateau, see figure 7(f). Vortices are less intense (the maximum of \( V_\theta \) is worth approximately 15\% of \( V_\infty \)) and more concentrated (transition to \( r^{-1} \) occurs at smaller \( r/b \)). Again, the decrease of the maximum of \( V_\theta \) at \( x/b = 9 \) results from change in the probing mesh size.

A single scale vortex model, e.g. the Rankine or the Lamb-Oseen vortex, is clearly not appropriate. As mentioned in §2.2, at least two characteristic scales must be used to model the flow. The idea behind representing vortices with a multiscale model was highlighted by Spalart (1998). When considering a law such as \( V_\theta \propto r^{-\alpha}, \alpha < 1 \), to represent the intermediate region, the classical model proposed by Betz (1932) leads to \( \alpha = 1/2 \) for a wing with an elliptic load. The Betz model is based on the principle that the vortex sheet rolls up in a cylindrical way around its centroid, assuming a local conservation of the vorticity moments. The value of \( \alpha = 1/2 \) is also obtained assuming a self-similar roll-up as in the Kaden spiral (see Saffman, 1992). A two scale model of the type:
may be adopted. The values of the constants are:

\begin{equation}
A = \frac{\Gamma}{(2\pi \eta_1 \sqrt{\eta_2})}, \quad B = \frac{\Gamma}{(2\pi \sqrt{r_2})}, \quad C = \frac{\Gamma}{2\pi}
\end{equation}

Our results suggest: \( \eta_1 / h = O(10^{-2}) \), \( r_2 / h = O(10^{-1}) \). A fourth region, corresponding to the plateau observed in figure 7(c) and 7(e), could also be added. As discussed above, the internal scale \( \eta_1 \) is poorly described by these mean flow measurements. All we can say up to this point is that it should be smaller than the probing mesh size, i.e. \( \eta_1 / h \leq 0.01 \). This topic will be discussed further below, in §5. On the other hand, the external scale \( r_2 \) is close to \( r = 0.1 b \).

As for the axial velocity component, the trailing vortices are usually observed to be of the wake type, i.e. with an axial velocity smaller than the free-stream velocity. Figure 8(a) shows the downstream evolution of the axial velocity deficit measured at the centre of the dominant vortices. The results corresponding to \( x/b = 0.5 \) and 1 correspond to the wing tip vortex before it merges with the flap vortex. At \( x/b = 0.5 \), a 22% deficit at the core of that vortex is obtained in the high lift case. This deficit sharply decreases downstream. After \( x/b = 3 \) the velocity defect is only sensible in the high lift case, where it stabilises at approximately 8%. Note that this value corresponds to nearly 30% of the maximum tangential velocity, and so it cannot be neglected.

Figure 8(b) shows the result of a cylindrical averaging performed on the scatter plots of the axial velocity deficit \( (V_\infty - U_{\min})/V_\infty \) in the vortex at \( x/b = 3, 5, \) and 9. As it can be observed in this figure, in the vicinity of the vortex centre, the axial velocity deficit can be fitted to a logarithmic law. A two-scale model may also be proposed to describe the axial velocity defect:

\begin{equation}
\begin{array}{ll}
r \leq r_1 : U = U_{\min}, & r_1 \leq r \leq r_2 : U = U_{\min} + A \log(r / r_1), \\
& r \geq r_2 : U = V_\infty,
\end{array}
\end{equation}

\( r_1 \) and \( r_2 \) are the internal and external scales of the axial velocity distribution. As for the tangential velocity, the inner scale \( r_1 \) is very small: for the three curves in figure 8(b), departure to the logarithmic law only occurs on distances comparable to the probing mesh size. From figure 8(b), \( r_2 = 0.07 \). Yet, we have no theoretical explanation for the presence of such a logarithmic law.

4.3. Fluctuations

Figure 9 shows distributions of the fluctuation energy rate \( \sqrt{k / U_\infty} \) at \( x/b = 0.5 \) and \( x/b = 9 \). The quantity \( k \), which characterises the kinetic energy of the fluctuations, is evaluated here using the expression
\[ k = (u^2 + 2w^2)/2 \]. The third velocity component \( v^2 \) is available but is not accounted for because it is affected by major errors, as discussed in §3.2.2. This approximation has no effect on the conclusions of our analysis.

As it can be seen in figure 9(a), for \( x/b = 0.5 \) the turbulence is distributed rather uniformly over the entire wake, with a maximum located in the fuselage area and concentrations in the wing-tip and flap-tip vortices. For \( x/b = 9 \), regular turbulence has decayed and a fluctuation peak of approximately 10% still remains within the vortex core. The downstream evolution of this peak energy has been plotted in figure 10 for the two configurations tested. The maximum of \( k \) decays in a first phase and remains almost constant afterwards. These perturbations do not characterise a developed turbulence but a meandering of the vortices. As mentioned in §3.2.1, this phenomenon has been observed by means of tomoscopic visualisations.

Hot wire measurements show that the vortex meandering corresponds to a broadband spectrum (see Devenport et al., 1996). This is illustrated in figure 11(a), where is plotted the energy density of the axial component of the velocity measured with a single hot wire set in the core of the vortex for \( x/b = 5 \). The particular nature of the fluctuation associated to vortex meandering is highlighted when comparing figure 11(a) with figure 11(b). The latter corresponds to the spectrum measured in the same streamwise plane, but in the “regular” wake, outside the roll-up region. Indeed, the spectrum of figure 11(a) exhibits a sharp energy excess for frequencies smaller than, say \( f \leq 1000 \text{ Hz} \) (red curve). This characterises meandering which is the most obvious manifestation of the unsteady aspect of the flow in the extended near field. The precise origin of these perturbations is still not well understood. This phenomenon is discussed below.

5. THE VISCOUS CORE RADIUS

Apart from the general limitation of 3D LDV measurements discussed in §3.2.2, there are other sources of uncertainties specific to LDV measurements of highly concentrated vorticity. As pointed out above, the internal core \( r_1 \) is too small to be described with the present LDV mesh size. The two point correlations that will be presented in §7 require a method to define the vortex centre with great precision. This has been done through an inspection of the velocity histogram when moving the LDV measurement volume within the core region. Figure 12 shows the histogram of the vertical velocity, \( W \), obtained at \( x/b = 5 \) when a one component LDV measurement volume coincide with the vortex axis, figure 12(a), and when it is shifted by a spanwise distance of half a millimetre, figure 12(b). A total number of \( 10^6 \) particles has been considered here.

The histogram in figure 12(a) exhibits a double peak distribution, which means that particles with positive and negative vertical velocities are seen by the measurement volume within this distance. This change of sign may result from a combination of three effects:

(i) the spatial integration of the LDV measurement volume,
(ii) the centrifugation of the particles,
(iii) the meandering of the vortex.
(i) The spatial integration is particularly important here. This is illustrated in figure 13 which depicts the measurement volume of a 1D fringe LDV set-up and the corresponding vertical velocity component map. This 1D fringe set-up has been used during the measurements of the two point correlations which will be discussed in §7. The characteristic length scale of the operating part of the measurement volume is \( l \approx 2 - 3 \text{mm} \) and is nearly twice that of the 3D LDV configuration in which this volume is reduced by intersection with the two other color beams. Figure 13 shows that this length scale is much larger than the internal core radius where sharp gradients occur. Particles crossing the right-hand side (respectively the left-hand side) of this fringe volume will contribute to negative (respectively positive) values of \( W \). These effects, considered alone, lead to a broadening of the velocity histogram and to a two peak distribution.

(ii) The double peak distribution is strongly accentuated if this spatial integration effect is combined with centrifugation of the seeding particles. The effect of centrifugation is illustrated in Figure 14 which shows the spatial distribution of the data rate corresponding to the result of figure 13. The mean number of seeding particles detected in the measurement volume decreases by a factor 5 in the vortex centre; this increases the respective contributions of the peak values to the histogram. Note that a strong correlation between the time intervals between two successive particles and the measured velocity component, which evaluates the so-called “velocity bias” in LDV, could also participate to a departure of the histograms from equilibrium (irrespectively from centrifugation). But this correlation coefficient has been found to be weak (not larger than 0.1).

(iii) At last, vortex meandering is also contributing to such histograms. This may be illustrated by considering the simple model problem of a Lamb-Oseen vortex subjected to a random motion around its centre. One considers the basic 2D velocity field:

\[
\begin{align*}
V_0(y, z) &= -\left(1 - \exp\left(-\frac{y^2 + z^2}{\rho^2}\right)\right) \frac{z}{y^2 + z^2} \\
W_0(y, z) &= \left(1 - \exp\left(-\frac{y^2 + z^2}{\rho^2}\right)\right) \frac{y}{y^2 + z^2}
\end{align*}
\]  

(13)

corresponding to a Lamb-Oseen vortex of axis \( Ox \) with a unit radius and circulation \( \Gamma = 2\pi \). Using a method analog to that of Devenport et al. (1996), one supposes that the vortex center is subjected to a movement characterized by a probability density \( P(y_c, z_c) \) of the vortex center position \( (y_c, z_c) \). Let:

\[
H(W, y, z) = \int_{R^2} \delta(W - W_0) P(y_c, z_c) dy_c dz_c
\]  

(14)

the probability of measuring a vertical velocity equal to \( W \) at position \((y, z)\). One has \( \int_R H(W; y, z) dW = 1 \). The mean field seen through temporal averages is:

\[
<W> = \int_{R^2} W_0(y - y_c, z - z_c) P(y_c, z_c) dy_c dz_c
\]  

(15)

The same holds for \(<V>\). The shear stress is:
\[ <v'w'> = \iint_{\mathbb{R}^3} V_0 (y - y_c, z - z_c) W_0 (y - y_c, z - z_c) P(y_c, z_c) dy_c dz_c - <V> <W> \]  

(16)

We have evaluated (14) and (16) in the case of an isotropic gaussian random motion, with rms amplitude \( \sigma \):

\[ P(y_c, z_c) = \frac{1}{2\pi\sigma} e^{-\frac{y_c^2 + z_c^2}{2\sigma^2}} \]  

(17)

Figure 15(a) and (b) shows the probability density \(14\) with \( \sigma = 1.5 \), evaluated respectively at the mean center position of the vortex, \( H(W;0,0) \), and at a distance of half a vortex radius from the mean center, \( H(W;0.5,0) \). This motion results in a double peak histogram which compares qualitatively well with that observed in the experiment, see figures 12(a)-(b). Figure 15(c) shows the corresponding shear stress distributions \(<v'w'>\), see (16). Comparing with figure 15(d) shows that there is a good correspondence between the model and the experimental results concerning this quantity.3

Coming back to the experimental histograms, figure 12(b) shows that a one peak histogram is retrieved when translating the measurement volume on a distance not larger than 0.5mm. One may conclude that, both the internal core radius and the meandering amplitudes are small, e.g. of the order of one millimetre. This is smaller than the characteristic scale of the measurement volume. Note that an internal core radius size of 1mm at \( x/b = 5 \) is comparable to the characteristic viscous diffusion length \( l_v = \sqrt{\nu x/V_{\infty}} = 0.8\text{mm} \). This confirms that \( r_1 \) is really a 'viscous radius'.

As pointed out by Devenport et al. (1996), meandering introduced a "bias" in the measurement of the mean velocity and vorticity presented above. Elimination of the meandering contribution, as proposed by Devenport et al. (1996), would necessitate to make a discrimination between meandering, particle centrifugation and spatial integration. This has not been undertaken.

The double peaked histograms discussed above were used to localise the vortex axis in the course of the two-point measurements presented in §7 bellow: the vortex centre were defined as the position which maximises the two peaks of the velocity histogram.

6. ON THE ORIGIN OF VORTEX MEANDERING

The origin of vortex meandering is not clear. A comprehensive analysis of the problem has been made by Devenport et al. (1996). These authors made a thorough investigation of the phenomenon behind the tip of a NACA 0012 half-wing and proposed a correction method for the mean velocity profiles. The method was based on a series decomposition of the velocity profiles and on the hypothesis of Gaussian form of the p.d.f. of vortex position similar to (17). Corrections were negligible outside the core region. The main impact of meandering on

3 Errors on \( v' \) discussed in §3.3.2. have no incidence on this qualitative result.
the velocity profiles was shown to be an overestimation of about 10-15% of the core radius and an underestimation by the same amount of the maximum tangential velocity and of the axial velocity deficit. Also, their analysis of velocity auto-spectra, measured in various position on the spiral wake centreline towards the vortex centre, revealed a progressive increase of the energy in the low frequency domain, due to the meandering. On the other hand, they observed a decrease of the high frequency contribution (Figure 21 of their paper). This may result from an inhibition by rotation of the energy cascade at smaller scale. Within the vortex core, the higher frequencies were observed to follow a $f^{-3}$ law (with $f$ the frequency) which suggested a complete blocage of the 3D energy cascade as in 2D turbulence (figure 22 of their paper). At last, Devenport et al. (1996) found that the meandering amplitude increases with the downstream distance and decreases with the angle of attack. Following these authors, the latter variation suggests that the most important source of meandering may be related to wind-tunnel perturbations, the contribution of which decreases as the strength of the vortex increases.

Some of the above conclusions are confirmed by the present experiment but our results depart from those of Devenport et al. (1996) on several respects. Following these authors and others, the origin of meandering could be attributed to several mechanisms such as:

(i) Interference with wind tunnel unsteadiness,
(ii) Excitation of perturbations in the vortex cores by turbulence contained in the wake,
(iii) Linear co-operative instabilities,
(iv) Propagation of unsteadiness originating from the model.

The results presented below will enable to discard point (i) and suggest that meandering is likely a superposition of the three other mechanisms.

6.1. Vortex meandering and wind tunnel interference

As suggested by Devenport et al. (1996), the meandering could originate from the receptivity of the vortices to perturbations within the wind tunnel. The F2 wind tunnel exhibits a particularly low residual turbulence level, of the order of $10^{-3}$. These weak perturbations are not expected to play a dominant role. However, an original test has been conducted in the course of our experimental program which enables to characterise the sensibility of the vortex to perturbations in its proximity. These tests consisted in considering a “half model configuration” obtained by adding a long splitter plate which separates the test section of the tunnel into two independent, almost identical, channels. This is shown in figure 16. The leading edge of the plate was located 1 span ahead of the model fuselage and its length was 8 spans. Two objectives were assigned to this experiment. The first was to evaluate the response of the vortex to perturbations emanating from a thick flat plate boundary layer located in its neighbourhood; the second was to evaluate possible differences in the results between a full model configuration and a half model configuration.

The splitter plate boundary layer has been probed with the hot wire in few positions downstream of the model, at the same vertical positions as that of the vortices. At $x/b=5$, the standard, displacement and momentum
thickness are found to be $\delta = 60\, \text{mm}$, $\delta_1 = 10.1\, \text{mm}$ and $\theta = 7.43\, \text{mm}$ respectively. This boundary layer is fully turbulent. It is also highly perturbed by the fuselage wake. Figure 17 shows the streamwise $U$ component profile at $x/b = 5$ with or without the splitting plate. Note that the mean position of the vortex has moved slightly away from the wall. The vortex center is located $175\, \text{mm}$ aside of the wall, that is at a distance of $115\, \text{mm}$ of the edge of the boundary layer. A question is to know if this affects the meandering properties. Figure 18 which compares co-spectra measured in the vortex center shows that this is not the case. The level and shape of the two spectra are very similar. There are slight differences, but they remain within the uncertainties due to determination of the vortex center.

It may be concluded from these tests that the vortex meandering is almost insensitive to wind tunnel wall effects. Another useful conclusion is that, hopefully, the use of a half model in spite of a full model in wind tunnel tests gives equivalent results with regards to vortex unsteady properties.

### 6.2. Vortex meandering and turbulence

Temporal direct simulations of an initially unstable $q$-vortex (or Batchelor vortex) have been performed by Ragab & Sreedhar (1995) and Sipp et al. (1998). The results of these numerical studies may be summed up as follow. After a linear growth regime, turbulence develops. The fully developed regime then leads to a gradual reduction of the axial velocity deficit, through turbulent transport of the axial momentum, whereas diffusion of the angular momentum is strongly inhibited. It turns out that turbulence only affects the axial momentum of the vortex, leaving the angular momentum practically unchanged. The result is that the vortex does not diffuse quicker than in a laminar regime. The linear stability of the $q$-vortex is characterised by the parameter $q \equiv 1.57 V_{\theta_{\text{max}}} / \Delta U$ (“swirl number”) where $V_{\theta_{\text{max}}}$ designates the maximum of $V_\theta$ and $\Delta U$ is the maximum axial velocity deficit (see Ragab & Sreedhar, 1995). Reduction of the axial velocity deficit and conservation of the tangential velocity profile lead to an increase of $q$. By this way, the vortex returns progressively to a stable linear regime. The elimination of instabilities then leads to re-laminarisation of the vortex thanks to dissipation. The conclusion of these numerical simulations is that turbulence may develop in a vortex under action of a strong axial shear, but this is only a transient state; isolated axisymmetric vortices, far away from their source, are laminar.

In the present experiment, the swirl number $q$ is close to the critical value $q=1.5$ in the wing tip and flap vortices at $x/b = 0.5$. Downstream, $q$ increases rapidly. When $x/b = 9$, we have seen that $V_{\theta_{\text{max}}}/V_\infty = 0.23$, $\Delta U/V_\infty = 0.08$, hence $q \approx 4.5$. Thus, if considering each vortex as an isolated $q$-vortex, small scale turbulence may be present but it has almost no effect on the vortex development; in the far-wake field, the vortices should evolve as if they were laminar. Trailing vortices differ from $q$-vortices from a transport of some turbulence by the wake roll-up and from possible contribution of short wavelength co-operative instabilities.

The velocity co-spectra gives information on this problem. Figure 19 shows spectral densities $S_{uu}(f)$ of the axial component of velocity measured at $x/b = 5$ when the hot wire is moved along a vertical line crossing the
vortex core. In figure 19(a), which corresponds to the high lift case, nine vertical positions have been considered with an increment \( \Delta z = 1\text{mm} \); in figure 19(b), the frequency contributions to the energy are highlighted through the product \( f \times S_{uu}(f) \) in the case where the probe is in the vortex centre. The clean case has been explored at six different positions with a step \( \Delta z = 2\text{mm} \), see figure 19(c) and (d). As reference, the frequency corresponding to a wavelength \( \lambda = 0.1\text{b} \), our evaluation of the vorticity region, is close to 1kHz \((1115\text{Hz})\). Considering the small scale activity, a \( f^{-3} \) behavior in the high frequency range is found here, in figure 19(a) and (c), as in Devenport et al. (1996). The statement that the small scales \((l < \lambda, f > 1\text{kHz})\) could become energetically inactive under the action of the strong rotation in the roll-up region is very appealing. Note that small scale energy increases when approaching the center in the high lift case (Fig.19(a)), whereas it remains almost constant in the clean case (Fig.19(c)). Small scale activity is much more pronounced in the high lift case, probably because of the higher turbulence level present in the wake of the high lift model.

The larger scales \((f < 1\text{kHz})\) behave very differently. When going towards the vortex center, the contribution of the low frequencies to the energy increases dramatically. This is the typical signature of meandering. A scaling law \( f^{-4/3} \) may be observed in the vortex center in the high lift case, see figure 19(a), but its origin is not yet understood. Accumulations of energy at three preferred frequencies are observed in the high lift case (arrows). These “peaks” could find their origin in the emergence of co-operative instabilities, as discussed below. The clean case core center, figure 19(c), is characterized by a smoother spectrum with no peak. Comparison between figures 19(b) and 19(c) shows that the small scales are much more energetic in the high lift case.

These spectra are suggesting that meandering and turbulence concern very different frequencies. As suggested by Miyazaki & Hunt (1999), a possible relation between turbulence and meandering of vortices is the excitation of a vortex by the turbulent perturbations contained in the layer during roll-up. According to their theory, weak perturbations in the external flow surrounding the vortex core are affected by a rapid distortion by the tangential shearing \( r \partial (V_\theta / r) / \partial r \). The perturbations are quickly axisymmetrised and reorganise themselves (through stretching) as vortex rings the amplitude of which grows linearly with time. Miyazaki & Hunt (1999) also show that while they are stretched and axisymmetrised, the external perturbations can excite growing asymmetric inertial waves into the vortex core. In the presence of intense turbulence of an atmospheric nature, this mechanism likely plays a major part in initiating a vortex meandering and co-operative instabilities (see Corjon et al., 1999, Gerz et al., 1998). However the theory developed by Miyazaki & Hunt (1999) remains limited to very idealised flows. Its extension to more realistic vortex models is the object of ongoing work.

As a conclusion, a relationship between turbulence contained in the wake and meandering cannot be discarded but it remains unknown. The separation between the scales involved in these two phenomena suggests that they are almost independent.
6.3. Vortex meandering and linear stability

Energy accumulations can be observed in figure 19(a). They are located at, approximately, \( f = 15 \text{Hz} \), \( 55 \text{Hz} \) and \( 400 \text{Hz} \). A question is to know if these peaks are associated to linear stability mechanisms.

The peak located around \( f = 15 \text{Hz} \) may correspond to the emergence of the Crow instability. The Crow instability is the long wave type of a well known family of linear “co-operative instabilities” which results from the interaction between the two vortices in a vortex pair (Crow, 1970). As known from Moore & Saffman (1971), the characteristics of the Crow instability only depend upon the internal structure of the vortex core through the effective core radius \( r_{\text{eff}} \), see (6). A rough evaluation is \( r_{\text{eff}} \approx 0.1b \). The theoretical wavelength for the Crow instability is \( \lambda_{\text{Crow}} \approx 8\tilde{b} \), \( (k\tilde{b} = 0.8) \). With \( \tilde{b} \approx 340 \text{mm} \), this gives \( f_{\text{Crow}} = V_{\infty}/\lambda_{\text{Crow}} \approx 18 \text{Hz} \), a value close to \( 15 \text{Hz} \). The amplification rate of the Crow instability is \( \sigma = 0.81\pi/2\pi\tilde{b}^2 \). Fabre et al. (2000) have shown that this kind of instability is essentially convective. So, its development takes place on a characteristic distance \( x = V_{\infty}/\sigma \). Using (4) and \( \tilde{b} = (\pi/4)b \), in the high lift case \( (C_L = 1.7) \) we find that the characteristic distance of instability development is \( x/b = 7.61 AR/C_L \approx 30 \). In conclusion, only the very first stages of the development of the Crow instability can be felt in the extended near field which is investigated in the present experiment. However, the vortices being very concentrated, vortex displacements of weak amplitude may generate strong fluctuations on a fixed hot wire probe. So there is no impossibility to establish a relationship between the first energy peak observed in the spectrum in figure 19(a) and the Crow instability even if the latter only emerges. However, as we will see from inspection of two-point correlations presented in §7, this conclusion is obscured by the existence of slight perturbations associated to model structure modes which affect frequencies close the Crow frequency.

As evidenced by figure 19(b) and (d), the main difference between the two wakes is the presence of a strong energy overshoot around \( f = 400 \text{Hz} \) in the high lift case. This energy accumulation could thus originate from the initial multipolar structure of the high lift model’s wake.

Various multipolar co-operative instabilities may be invoked. A configuration including two pairs of co-rotating vortices, such as the tip and flap vortices of our experiment, has been studied by Crouch (1997). This model is affected by an instability characterised by wavelengths scaling with the separation \( d \) between the co-rotating vortices, \( \lambda_{\text{Crouch}} = 6d \) \( (kd = 1) \) and by a characteristic time scale comparable to that of the Crow instability. In our experiment, \( d = 35 \text{mm} \), corresponding to \( f = 230 \text{Hz} \) which falls between the observed peaks at \( f = 55 \text{Hz} \) and \( f = 400 \text{Hz} \). The amplification rate of these instabilities are of the same order than that of the Crow instability (Crouch, 1997); this means that the characteristic distance of their development is again \( x/b = 30 \). In the experiment, the flap and wing tip vortices merge within a distance of \( x/b = 2 \), which fixes a limit to the contribution of the co-rotative instability proposed by Crouch.
Another configuration, involving internal vortices of opposite sign, has been studied by Rennich & Lele (1999) and by Fabre & Jacquin (2000). In our experiment, internal vortices of opposite sign to that of the wing and flap vortices are generated by the inner flap edge close to the fuselage, see figures 5(a) and 5(b). From these figures, a value of $d = 45\text{mm}$ may be adopted for the internal vortex separation. As shown by Fabre & Jacquin (2000), for this configuration, the most amplified linear mode corresponds to a short wavelength instability which scales with the internal vortex separation $d$ ($kd = 0.8$). The corresponding frequency is $f = 140\text{Hz}$. The characteristic time scale of this instability is nearly twenty times that of the Crow instability, but this short wavelength mode does not affect significantly the outer vortices. Longer wavelength instabilities also develop on wavelengths similar to the Crow instability, with a time scale ten times larger to that of the latter; so, these modes could contribute to the first energy accumulation around $f = 15\text{Hz}$. Note that the stability analysis of Fabre & Jacquin (2000) supposed that the four vortex configuration remains stationary, an hypothesis which is not fulfilled in the experiment. It cannot be excluded, however, that long wavelength instabilities of this type may contribute to the low frequency part of the vortex meandering. Further support to this hypothesis will be given when considering the two-point correlations. But, this type of co-operative instability is unlikely to contribute to the energy overshoot identified around $f = 400\text{Hz}$.

Finally, one may be tempted to associate this accumulation of energy to short-wave co-operative instabilities (Widnall-type instabilities, Moore & Saffman, 1975, Tsai & Widnall, 1976). The frequency $f = 400\text{Hz}$ corresponds to convection of a wavelength $\lambda = 0.28b$. For the case of a Lamb-Oseen vortex of radius $a$, the short-wave co-operative instabilities correspond to wavenumber $ka = 2.2$ (see Sipp et al., 1998), i.e. $\lambda = 2.7a$. This would correspond to the observation if considering that the vortex is equivalent to a Lamb-Oseen vortex with radius $a = 0.1b$ close to the dispersion radius $r$ of our experiment. But contrary to the Crow instability, the short-wave instabilities do depend on the internal structure of the vortex core. For the more suited two scale model presented in §4.2, see (10), preliminary results have shown that the Widnall instability scales on the internal core radius $r_1$, not on $r_2$ (Fabre & Jacquin, in preparation). This would mean that this peak in the spectra is not associated to a linear amplification of a Widnall-type instability. Further investigations are needed to explain this accumulation of the energy around $f = 400\text{Hz}$.

Considering now the clean case shown in figure 19(c) and 19(d), meandering is also evidenced but peaks of energy are absent. The characteristic distance of the instability growth is larger than that of the high lift case. An estimation is $x/b = 70$ for $C_L = 0.7$. The absence of vortex merging and a slower development of the long wave co-operative instabilities could explain the differences. Also, the trailing wake of the clean model contains less turbulence than that introduced by the high lift devices which means smaller initial perturbations of the linear instability mechanisms.

6.4. Conclusions

The above discussion leads to the following conclusions about the vortex meandering:
- In our experiment, vortex meandering seems independent of wind tunnel perturbation effects.
- Its relationship with turbulence remains unknown.
- The various co-operative instabilities described in the literature affect the same range of frequencies than meandering. These instabilities may thus contribute to meandering but detailed analysis does not show a clear relationship between the two phenomena.
- Adaptations of this stability analyses to more realistic situations, e.g. mean velocity profiles such as those presented in §4.2, are underway. But preliminary results show that this may be not decisive. A clear distinction between the high lift and the clean cases is made through figures 19(c) and 19(d), which strongly suggests that the energy accumulations in the spectra in the high lift case are related to the multipolar nature of the initial wake. Answers could be found in a detailed analysis of the vortex flow which results from the merging of the wing and flap tip vortices. But a complete theory of the vortex merging at high Reynolds numbers is not yet available.

In the following section other aspects of the vortex meandering are addressed by means of two point measurements

7. Two-point correlations

The vortex meandering has been also characterised by means of two-point correlations using the LDV technique shortly described in §3.2.2.

7.1. Post-processing

The LDV set-up enables to determine the space-time correlation $R_{ww}(\xi_1, \xi_2, \tau) = w'(\xi_1, t)w'(\xi_2, t+\tau)$ and the correlation coefficient:

$$C_{ww}(\xi_1, \xi_2, \tau) = \frac{w'(\xi_1, t)w'(\xi_2, t+\tau)}{\sqrt{w^2(\xi_1, t)w^2(\xi_2, t)}}$$

where $w'$ denotes the vertical velocity fluctuation. A sample length of $10^6$ values has been considered for each measurement point. Applying a FFT algorithm to the two auto-correlation functions $R_{ww}(\xi_1, \xi_1, \tau)$, $R_{ww}(\xi_2, \xi_2, \tau)$ and to the cross correlation function $R_{ww}(\xi_1, \xi_2, \tau)$ leads to the energy spectra $E_{ww}(\xi_1, \xi_1, f)$, $E_{ww}(\xi_2, \xi_2, f)$ and to the cross-spectrum $E_{ww}(\xi_1, \xi_2, f)$, respectively. As LDV provides random sampling of the signals, the correlations are computed using a slotting technique. The frequency range $\Delta f$ and frequency resolution $f_{max}$ are set by choosing the maximum time lag, $\tau_{max}$ and the time step, or bin width, $\Delta \tau$. One has $f_{max} = 1/\Delta \tau$ and $\Delta f = 1/\tau_{max}$.
In the following, \( C_{ww}(x_1, x_2, \tau) \) will denote the correlation coefficients obtained when the two measurement volumes are set at distances \( x = x_1 \) and \( x = x_2 \), their exact position in these planes being specified otherwise.

The technique is illustrated in figure 20. Figures 20(a) and 20(b) show the correlation coefficients \( C_{ww}(x_1/b, x_2/b, \tau V_w/b) \) and their corresponding spectral densities obtained for \( x_2/b = 5 \) and \( x_1/b = 3 \). The two measurement volumes are separated by a distance \( \Delta x = 2b \) and they are located at the center of the left hand side vortex. These positions were determined using the 'double peak histogram' technique described in §5. \( \tau_{\text{max}} \) and \( \Delta \tau \) have been set to obtain a frequency range of 1kHz and a frequency resolution \( \Delta f = 0.5Hz \). Figure 20(c) shows the correlation curves obtained when limiting the frequency range to \( f_c = V_w/\Delta x = 56Hz \) corresponding to the measurement points separation, by means of a numerical low pass filter. Removing by this way the contributions of the lengthscales smaller than \( \Delta x \) leads, logically, to an increase of the coefficients \( C_{ww} \).

The LDV spectrum \( E_{ww}(x_2, x_2, f) \) in figure 20(b) (blue curve) and the hot wire spectrum \( S_{uu}(f) \) of figure 19(a) (red curve) look very different. These differences may be explained as follows. First, the LDV spectrum is an energy spectrum of the vertical velocity based on a measurement subjected to a large spatial integration effect and to particle centrifugation, as discussed in §5. The second spectrum is a spectral density deduced from a hot wire signal which is free from particle effects. This measure is also subject to a spatial integration effect, the wire being 1mm long, but hot wire integrates spatially the velocity fluctuations differently than LDV. As evidenced by figure 20(b), the main limitations of spectral analysis by a LDV system are a strong reduction of the bandwidth and a low signal to noise ratio. One notes on the blue spectrum of figure 20(b) a sharp decrease of the energy beyond \( f = 200Hz \) which is not observed in the hot wire spectra. The signal is then overwhelmed by the LDV noise.

In conclusion, these two-point LDV measurements are subjected to tremendous limitations when the inner region of the vortex is explored. This makes the meaningful part of the LDV information limited to low frequencies. But this is sufficient for our purpose which is to characterize statistical links between the vortex core movements felt at different locations along the vortices, separated by large distances.

### 7.2. Correlation coefficients

Figure 21 shows the correlation \( R_{ww}(x_1 + \Delta z \xi_x, x_2 + \Delta z \xi_x, \tau) \) obtained when the two measurement points are separated by a distance \( \Delta x = 0.22b \) (\( x_1/b = 4.78 \), \( x_2/b = 5 \)) and when they are shifted altogether from the vortex centre by a small vertical distance \( \Delta z \). The cross correlation falls down rapidly as the measurement points leave the vortex centre, see figure 21(a). But the correlation coefficient (obtained after normalisation of \( R_{ww} \) by the r.m.s. of the two signals) remains almost constant even when \( \Delta z \) exceeds the vortex core width, as it is the case when \( \Delta z = 5mm \). This proves that the perturbations which contribute to the correlation correspond to solid body movements of the vortex.
All the results presented from now have been obtained by setting the downstream measurement volume fixed in the centre of the vortex at \(x_2/b = 5\), see figure 4. Figure 22 shows the variation of the coefficient \(C_{\infty} \left( x_1/b, x_2/b, \tau_{\infty}/b \right)\) when the moving point \(x_1\) is set at various longitudinal positions upstream \(x_2/b = 5\), in the centre of the same vortex. The double peak histogram method described in §5 is used for the determination of the vortex centres. Figure 22(a) shows the results obtained for \(x_1/b \geq 2\), that is for separations \(\Delta x/b \leq 3\). The position \(x_1/b = 2\) corresponds to the end of the wing tip/flap vortex merging. Figure 22(b) shows the results obtained when the point \(x_1\) is set further upstream in the wing tip or along the flap tip vortex, at \(x_1/b = 1\) and \(x_1/b = 0.01\).

These figures confirm that a significant correlation level is maintained on distances which scale with the model span. For instance the cross-correlation coefficient is 0.3 when the two measurement points are separated by a distance of two wing spans. They also show that the optimal time delay where \(C_{\infty}\) is maximum is exactly equal to the convection time, \(\tau_{\infty} = \Delta x/V_{\infty}\). This proves that the low frequency perturbations associated to meandering are convected at the free-stream velocity. In figure 22(b), where distinction is made between the wing tip and the flap tip vortices, the correlation decreases in the same line of the preceding ones in the wing tip vortex whereas it becomes erratic in the flap vortex. This could be the mark of dynamic stall on the flap. The correlation vanishes when the upstream point is set close to the model at \(x_1/b = 0.01\), that is near the “origin” of the two vortices.

Figure 23 details the case where the upstream measurement point is set in the plane \(x_1/b = 0.5\). Various positions in the wake are considered, as shown in figure 23(a). The wake is characterised in this figure through iso-contours of the axial velocity \(U/V_{\infty}\). When the moving point is set in the centre of the wing tip and flap tip vortices, see figure 23(b), a weak correlation is maintained in the line of the results of figure 22(b). The correlation almost vanishes when the moving point is set in the centroid of these two vortices (point labelled 22.2). Figure 22(c) shows that correlation vanishes when the upstream measurement point is set into the wake away from the main vortices but, interestingly, a significant negative correlation is obtained when considering the weak nacelle vortex (point labelled 23.3) and the “fuselage vortex”\(^4\) (point 23.4). The circulation of these two vortices being opposite to that of the trailing vortices, see figure 5(a), this change of sign of the correlation \(C_{\infty}\) indicates that the trailing vortices at \(x_2/b = 5\) and the secondary vortices of opposite sign at \(x/b = 0.5\) are displaced in phase along \(Oy\). For instance, horizontal displacements of the vortex centres towards positive \(y\), leads to positive fluctuations \(w\) when \(\Gamma < 0\) (trailing vortex) and to negative fluctuations \(w\) when \(\Gamma > 0\) (“fuselage vortex”). Note that the higher magnitude of \(|C_{\infty}|\) at \(x_1/b = 0.5\) is obtained within the “fuselage vortex”.

\(^4\) We think that this positive vorticity comes from the flap tip in the region of the wing-body junction.
Table 2(a) and (b) sums up the results on the correlations by giving for each case the maximum value of the correlation coefficients described above. The values obtained for both the unfiltered and the low pass filtered signals are given. The cut-off frequencies corresponding to the probe volume separation are indicated.

### Table 2(a) and (b)

<table>
<thead>
<tr>
<th>Moving point position</th>
<th>$x_1/b$</th>
<th>5</th>
<th>4.78</th>
<th>4.11</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>0.01</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downtstream point</td>
<td>$x_2/b$</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4.99</td>
<td>4.99</td>
</tr>
<tr>
<td>Axial separation</td>
<td>$\Delta x/b$</td>
<td>0</td>
<td>0.22</td>
<td>0.89</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>22.4</td>
<td>22.4</td>
</tr>
<tr>
<td>Probe separation “cut-off frequency” (Hz)</td>
<td>$f_c = \frac{V_m}{\Delta x}$</td>
<td>$\infty$</td>
<td>500</td>
<td>125</td>
<td>56</td>
<td>37.2</td>
<td>28</td>
<td>28</td>
<td>22.4</td>
<td>22.4</td>
</tr>
<tr>
<td>Correlation coefficient at optimal delay (all scales)</td>
<td>$C_{WW}(\xi_1, \xi_2, \tau_{max})$</td>
<td>1</td>
<td>0.61</td>
<td>0.5</td>
<td>0.34</td>
<td>0.18</td>
<td>0.12</td>
<td>±0.08*</td>
<td>≈ 0*</td>
<td>≈ 0*</td>
</tr>
<tr>
<td>Correlation coefficient at optimal delay, low pass filtering at $f_c$</td>
<td>$C_{WW,f_c}(\xi_1, \xi_2, \tau_{max})$</td>
<td>1</td>
<td>0.85</td>
<td>0.80</td>
<td>0.65</td>
<td>0.44</td>
<td>0.42</td>
<td>0.23*</td>
<td>±0.08*</td>
<td>≈ 0*</td>
</tr>
</tbody>
</table>

* Presence of a structure mode

### Table 2(b)

| Moving point position | $x_1/b$ | 0.5 | 0.5 | 0.5 |
| Downtstream point     | $x_2/b$ | 5 | 5 | 5 |
| Axial separation      | $\Delta x/b$ | 4.5 | 4.5 | 4.5 |
| Probe separation “cut-off frequency” (Hz) | $f_c = \frac{V_m}{\Delta x}$ | 24.8 | 24.8 | 24.8 |
| Correlation coefficient at optimal delay (all scales) | $C_{WW}(\xi_1, \xi_2, \tau_{max})$ | 0.06 | ±0.07 | -0.12** |
| Correlation coefficient at optimal delay, low pass filtering at $f_c$ | $C_{WW,f_c}(\xi_1, \xi_2, \tau_{max})$ | 0.26 | 0.37 | -0.5** |

** High coherence, see figure 24

**Table 2:** Summary of the maximum values of the correlation coefficient obtained when the fixed measurement volume is set at $x_2/b = 5$ and the moving measurement volume, at different values of $x_1/b : (a)$ all positions except $x_1/b = 0.5$, (b) $x_1/b = 0.5$ for the wing tip vortex, the flap vortex and the "fuselage vortex". The merging of the wing tip and flap vortex is located at $x/b = 2$.

Note in table 2(b) that at $x_1/b = 0.5$, the filtered correlation coefficient reaches a value of -0.5 in the "fuselage vortex". The special role of this vortex may be illustrated also by considering the coherence and phase spectra, according to the following relations:
\[ \gamma_{ww}(\vec{x}_1, \vec{x}_2, f) = \frac{|E_{ww}(\vec{x}_1, \vec{x}_2, f)|}{\sqrt{E_{ww}(\vec{x}_1, \vec{x}_1, f) E_{ww}(\vec{x}_2, \vec{x}_2, f)}} \]  

(19)

\[ \Phi_{ww}(\vec{x}_1, \vec{x}_2, f) = \tan^{-1} \left[ \frac{\text{Im}\{E_{ww}(\vec{x}_1, \vec{x}_2, f)\}}{\text{Re}\{E_{ww}(\vec{x}_1, \vec{x}_2, f)\}} \right] \]  

(20)

Figure 24 shows the coherence \( \gamma_{ww}(f) \) and the phase spectra \( \Phi_{ww}(f) \) corresponding to the results of figure 23(c) away from the two main vortices. A reference level \( \gamma_{ww} = e^{-1} \) is indicated bellow which coherence is usually considered as negligible. The phase spectra have been plotted against the frequency ratio \( f/f_c \) where \( f_c = V_o/\Delta x \). We have considered a frequency range limited to \( f \leq 250\text{Hz} \) beyond which the signal to noise ratio is too small at \( x/b = 5 \) (see Fig.20(b)). The positions of the moving probe are labeled according to figure 23(a). The figure shows that the perturbations inside the ”fuselage vortex” (point labeled 23.4) are the only ones which exhibit a significant coherence. In this case, the phase is linear up to \( f/f_c = 10 \). The slope is equal to unity which means that these perturbations are convected at the free-stream velocity. The initial value of the phase is \(-\pi\) in accordance with the negative sign of the correlation observed in figure 23(c). All other positions of the moving probe lead to a weak coherence and to more erratic phase distributions. Note in particular that the signal measured at point labeled 23.5, which is located very close to the ”fuselage vortex”, in the symmetry plane, is completely independent of the signal measured at \( x_1/b = 5 \).

At last, figure 25 shows the coefficient \( C_{ww} \) when the upstream point \( x_1 \) is set very close to the wing tip and flap tip origins, at \( x_1/b = 0.01 \). The data is the same as that shown in figure 22(b), but we consider here larger values of the normalised delay \( V_o \tau/b \). \( C_{ww} \) exhibits oscillations on a wavelength \( f = 13\text{Hz} \). Tests performed with an accelerometer set in the model have shown that this wave corresponds to a model structure mode. As shown by figure 25, this structure mode propagates into the wake and leads to a correlation oscillation of small amplitude (\( C_{ww} \leq 0.02 \)). This periodic component may be identified in the auto-spectrum and cross-spectrum of figure 20(b). Note that this frequency is not far from that of the Crow instability, as was mentioned in §6.3.

7.3. Conclusions

To sum up, these two-point measurements have shown that:

- The low frequency perturbations which contribute to the meandering felt at \( x_2/b = 5 \) exhibit a non negligible correlation with perturbations measured in the near field.

\[ ^5 \text{Phase analysis of the whole set of measurements discussed in this chapter has shown that the frequency ranges where coherence remains significant concern perturbations which are also convected at exactly the free-stream velocity.} \]
- The correlation coefficients, once low passed filtered at the "cut-off frequency" which corresponds to the probe volume separation $\Delta x$, are maintained at a level larger than 0.45 downstream the merging ($\Delta x \leq 3$, see table 2(a)).

- When progressing upstream along the individual vortices, a smaller correlation is maintained with erratic behaviors, especially in the flap vortex. This could be the mark of dynamic stall on the flap.

- The correlation falls to zero when the moving measurement volume approaches very close to the model. This suggests that meandering is not produced directly from surface flow unsteadiness. Co-operative interactions between the rolling-up vortices could then play a dominant role in its development. Note however that the measurements close to the model have only been performed at two positions (origin of the wing tip and flap vortices, see Fig. 22(b)). More data is necessary to confirm this finding.

- It is found however that a part of the correlated perturbations propagating in the wake correspond to a structure mode of the model. Its contribution to the meandering amplitude is found to be weak. On the other hand, its possible relationship with linear instabilities (Crow instability) cannot be excluded.

- A result of special interest is the identification of a significant negative correlation between meandering of the final vortices and the perturbations coming from the opposite signed vorticity located in the fuselage wake. This confirms that the presence of vorticity of opposite sign may enhance the vortex wake destabilization, as suggested by Rennich & Lele (1998), Fabre & Jacquin (2000) and Stuff (2000). This effect may be even larger for a real aircraft due to the additional counter-rotating vorticity coming from the horizontal tail (which is not present in our experiment).

8. CONCLUSIONS

An experimental data base on the formation and evolution of the vortex wake in the extended near-wake field of a small aircraft model for both clean and high lift configurations has been presented. The data is mainly based on 3D LDV measurements but it also contains information on the unsteadiness of the wake obtained by means of hot wire and 2 point 1D LDV measurements. The analysis of these results has led to the following main conclusions.

- Distribution of the tangential velocity $V_\theta$ within the vortex should be characterized at least by two scales. The first scale is the viscous core radius $r_1$. Its extend is essentially fixed by viscous diffusion. The second scale, $r_2$, characterizes the roll-up of the wake. As predicted by available theoretical analyses, a power law $V_\theta \propto r^{-\alpha}$ with $\alpha = 1/2$ is obtained for $r_1 \leq r \leq r_2$. The initial multipolar structure of the near-wake field of the model in a high lift configuration leads to the presence of a tangential velocity plateau beyond $r_1$.

- LDV measurements show that the unsteady properties of the wake are dominated by a meandering of the vortices. This phenomenon has been scrutinised more deeply by hot wire measurements and by means of a two-
point one-component LDV set-up. The results point out that meandering has not a unique origin and is likely the superposition of several mechanisms.

- Introduction of a flat plate which materialises the symmetry plane of the flow and which transforms the model into a half model, does not change significantly the meandering properties in spite of the presence of a very thick and turbulent boundary layer in the vicinity of the trailing vortex. This, together with the low level of background turbulence in the tunnel, indicate that meandering is likely independent of residual perturbations in the wind tunnel contrarily to what was suggested in some previous studies.

- A clear distinction between the high lift and the clean cases is made with energy accumulations in the spectra in the high lift case which are probably a remnant of the initial multipolar structure of the wake.

- Using the theory of linear stability, it is found that various co-operative instabilities affect the same range of frequencies as meandering. But the available theoretical results do not provide decisive interpretation of the observations. Answers could be found in a detailed analysis of the vortex merging mechanism.

- Analysis of hot wire spectra confirms the results of Devenport et al. (1996) concerning the 2D like behaviour of the smaller scale perturbations within the vortex (scaling law $f^{-3}$).

- The two-point correlation results show that the low frequency contributions to vortex meandering is also due to propagation of unsteady features which develop in the near-wake field. In particular, significant correlations are detected between the vortex perturbations at $x/b = 5$ and position in the vortices up to $x/b = 0.5$, especially in the wing tip vortex and in the “fuselage vortices”. Analysis of phases points out that the main part of the low frequency contribution to vortex meandering at $x/b = 5$ ($f \leq 250Hz$) is convected at exactly the free-stream velocity.

- Inspection of the field very close to the model does not show that correlation is maintained there. This seems to indicate that meandering of the vortices originates in the very first stage of the roll-up process in the nearfield behind the model. This must be corroborated by additional measurements.

- Low frequency model structure modes have been detected which may participate to the vortex meandering.

- A result of special interest is the identification of a coherent contribution to the meandering coming from the opposite sign vorticity generated in the fuselage region. The underlying mechanisms may be the counter-rotating vortex instabilities described by Rennich & Lele (1998) and by Fabre & Jacquin (2000).

**Acknowledgments**

A large amount of the experimental work has been conducted within the European project Eurowake No BE95-1085. The authors gratefully acknowledge the support that the Service des Programmes Aéronautiques (SPAé) has provided for developing the two-point correlation method. They wish also to express their gratitude to the F2 team and D. Soulevant, from ONERA, for their participation to the experiments, to M. Maurel and J.J. Degeilh, from EADS Airbus S.A. Centre de Toulouse, for their contribution to this research program and to D. Sipp, from SPAé, for fruitful discussions.
9. BIBLIOGRAPHY

3 Condit P.M. & Tracy P.W. : "Results of the boeing company wake turbulence test program ", in Aircraft wake turbulence and its detection, Plenum, 1971
4 Corjon A., Darracq D., Champagneux S. & Laporte F. : "Wake roll-up simulation up to the far-field", AIAA 99-3201, 1999
8 Donaldson C.P. & Bilanin A.J., "Vortex wakes of conventionnal aircraft", AGARDograph 204, 1975
21 Sipp D., Coppens, F. & Jacquin, L.: "Theoretical and numerical analysis of the dynamics of wake vortices". In Third International Workshop on Vortex Flows and Related Methods, Toulouse, France,


24 Stuff R.: Private communication


Figure 2 - A300 model (scale 1:100) in a « high lift » configuration in the test section of the F2 wind tunnel at ONERA Le Fauga Mauzac centre

Figure 3 - LDV system: (a) displacement system, (b) horizontal laser beam arrangement, (c) vertical laser beam arrangement
Figure 4 - LDV arrangement for two-point correlations

(a) Moving
(b) fixed (x/b = 5)

\[ \frac{\Omega_x (b/2)}{V_{in}} \]

Legend:
- 50.0
- 25.0
- 16.0
- 11.0
- 6.0
- 3.4
- 2.4
- 1.4
- 0.4
- 0.0
- -0.6
- -1.6
- -2.6
- -3.6
- -7.0
- -12.0
- -17.0
- -30.0
- -55.0
Figure 5 - Axial component of the vorticity $\Omega x b/2/\nu$ at $x/b = 0.5$ (a), 1 (b), 3 (c), 5 (d) et 9 (e) in half planes downstream of the model in a high lift configuration. The position of the wing is indicated by the dotted line. The vorticity reduction between figures (d) and (e) is due to a larger mesh size (from 2.5mm to 5mm).

Figure 6 - Circulation evaluated by azimuthal averages on contours centred on the vortex.
Figure 7 - Tangential velocity $V_\theta/V_\infty$ versus the radius centred on the vortex: (a) high lift configuration, $x/b=5$, scatter plot, (b) clean configuration, $x/b=5$, scatter plot, (c) high lift configuration, cylindrical averaging, (d) clean configuration, cylindrical averaging, (e) and (f) log-log plots of figures (c) and (d).
Figure 8 – Axial velocity: (a) Downstream evolution of the peak values of the velocity deficit in the wing tip vortex (in the high lift case, merging with the flap vortex occurs around $x/b = 2$), (b) log-lin plot of the cylindrical averaging of the velocity deficit in the high lift case.

Figure 9 - Turbulence rate $\sqrt{k}/V_\infty$ : (a) $x/b = 0.5$ and (b) $x/b = 9$
Figure 10 - Downstream evolution of the peak values of $\sqrt{k/V_\infty}$ in the wing tip vortex (in the high lift case, merging with the flap vortex occurs around $x/b = 2$).

Figure 11 - Spectral densities of the axial component of velocity at $x/b = 5$ in (a) the vortex center and (b) the fuselage wake. The red and green curves correspond to 20kHz and 2kHz sampling, respectively.
**Figure 12** - Histogram of the vertical velocity component $W(x_c/b = 5, y_c + \Delta y, z_c)$ measured by a 1D fringe set-up:

(a) $\Delta y = 0$. (b) $\Delta y = 0.5\text{mm}$. $(x_c, y_c, z_c)$ are the core centre co-ordinates.

**Figure 13** - Iso-values of the vertical velocity component measured by a 1D fringe set-up at $x/b = 4.11$. Sketch of the measurement volume. The volume obtained with the three component LDV set-up is nearly two times smaller.

**Figure 14** – Data rate (number of particles per second) corresponding to the measurements of figure 13.
Figure 15 - Random motion of a Lamb-vortex: (a) probability density of the vertical velocity, see relation (8), at \((y, z) = (0, 0)\), and (b) at \((y, z) = (0.5, 0)\). (c) iso-values of \(- \langle v' w' \rangle (y, z)\), see relation (10), (d) iso-values of \(- u' w'(x, z)\) measured at \(x/b = 5\). The levels shown at the figures are arbitrary; the circles shown in figure (c) are iso-values of the vorticity of the Lamb-Oseen vortex.

Figure 16 - "Half model configuration" obtained by sealing the model of figure 1 into a vertical wood flat plate which separates the F2 wind tunnel test section in two independent channels: upstream view showing the model, the separating plate and the hot wire displacement system.
Figure 17 - Hot wire profile of the longitudinal velocity across the vortex of the high lift configuration at $x/b = 5$ with the splitting plate (full line) and without the splitting plate (broken line).

Figure 18 – Spectral densities of hot wire fluctuations at $x/b = 5$ in the center of the vortex of the high lift configuration at $x/b = 5$ in the presence of the splitting plate (left plot) and without splitting plate (right plot). The red and black curved lines correspond to 20kHz and 2kHz sampling, respectively.
Figure 19 - Spectral densities obtained by hot wire of the axial velocity component at $x/b = 5$ along a vertical line crossing the vortex core. High lift case: (a) log-log plot of $S_{uu}(f)$ for $y=-172\text{mm}$, $z=-34$ to $-42\text{mm}$, $\Delta z = 1\text{mm}$, (b) lin-log plot of $f \times S_{uu}(f)$ at $y=-172\text{mm}$, $z=-38\text{mm}$. Clean case: (c) log-log plot of $S_{uu}(f)$ for $y=-188\text{mm}$, $z=-18$ to $-8\text{mm}$, $\Delta y = 2\text{mm}$, (d) lin-log plot of $f \times S_{uu}(f)$ at $y=-188\text{mm}$, $z=-14\text{mm}$. The red and black curved lines correspond to 20kHz and 2kHz sampling, respectively. As a reference, the frequency based on the dispersion radius is $f = V_\infty/ r \approx 1100\text{Hz}$.
Figure 20 - Correlation coefficient $C_{ww}(x_1/b, x_2/b, \tau V_\infty/b)$, (a), and energy spectra, (b), for $f \leq 1000Hz$: blue curves : $(x_1/b, x_2/b)=(5.5)$, green curves : $(x_1/b, x_2/b)=(3.3)$, red curves : $(x_1/b, x_2/b)=(3.5)$. Figure (c) : filtered correlation coefficient $C_{ww}(x_1/b, x_2/b, \tau V_\infty/b)$ for $f \leq f_c$ with $f_c = V_\infty/\Delta \nu = 56Hz$.

Figure 21 - (a) Correlation $R_{ww}(x_1 \varepsilon_{x_1} + \Delta z \varepsilon_{x_1}, x_2 \varepsilon_{x_1} + \Delta z \varepsilon_{x_1}, \tau)$ and (b) correlation coefficient for $x_1/b = 4.78$, $x_2/b = 5$ and $\Delta z = 0, 2, 5\text{mm}$.
Figure 22 - Correlation coefficient $C_{ww}(x_1/b, x_2/b = 5, \tau V_\infty/b)$: (a) moving point at $x_1/b = 0.22, 0.89, 2, 3$, $x_2/b = 0.22, 0.89, 2, 3$.

(b) moving point at $x_1/b = 1, 0.01$. 

(a) $C_{ww}$ with $x_1/b = 5$, $\tau V_\infty/b = 0$.

(b) $C_{ww}$ with $x_1/b = 5$, $\tau V_\infty/b = 0.22$. 

- $x_1/b = 4.78, x_2/b = 5, \tau V_\infty/b = 0.22$
- $x_1/b = 4.11, x_2/b = 5, \tau V_\infty/b = 0.89$
- $x_1/b = 3, x_2/b = 5, \tau V_\infty/b = 2$
- $x_1/b = 2, x_2/b = 5, \tau V_\infty/b = 3$
Figure 23 - Correlation coefficient $C_{ww}\left(\frac{x_1}{b} = 0.5, \frac{x_2}{b} = 5, \frac{r}{V_{\infty}}/b\right)$: (a) locations of the moving point within the wake characterised by iso-contours of $U/V_{\infty}$ (the corresponding iso-contours of the vorticity are shown in figure 5(a)), (b) moving point in the wing tip vortex, the flap tip vortex and their centroid, (c) moving point at various positions within the wing/flap wake and fuselage wake.
Figure 24 - Coherence $\gamma_{ww}(\Delta_1, \Delta_2; f\leq 250\,\text{Hz})$ with $x_2/b = 5$ and the moving point at $x_1/b = 0.5$ in the five locations corresponding to figure 23(c); (b) phase spectra $\Phi_{ww}(\Delta_1, \Delta_2; f\leq 250\,\text{Hz})$. The frequencies are normalised by $f_c = V_\infty/\Delta x$ in figure (b).

Figure 25 - Correlation coefficient $C_{ww}(x_1/b = 0.01, x_2/b = 5, V_\infty \tau/b)$