Paths of stable configurations resulting from the interaction of two disks falling in tandem

N. Brosse $^{a,b}$, P. Ern $^{a,b,*}$

$^a$ Université de Toulouse, INPT, UPS, IMFT (Institut de Mécanique des Fluides de Toulouse), Allée Camille Soula, F-31400 Toulouse, France
$^b$ CNRS, IMFT, F-31400 Toulouse, France

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**Abstract**

We investigate the interaction of two identical disks falling in tandem in a fluid at rest, at Reynolds numbers Re varying between 80 and 300. Wake visualization with fluorescent dyes was used to capture the interaction process: the trailing body accelerates until it catches up with the leading body. Thick disks ($\zeta = \text{diameter/thickness} = 3$) then lose their initial wakes, separate laterally and eventually fall side by side. In contrast, the wakes of thinner disks ($\zeta \geq 6$) merge into a single wake and the bodies continue their fall together, adopting a Y-configuration. The paths associated with this stable configuration were investigated in detail by three-dimensional trajectography. Three regimes were identified. At the lowest Re the Y-configuration falls along a rectilinear non-vertical path, but at higher Re the centre of gravity of the pair describes a periodic path contained in a plane slightly tilted relative to the vertical, and the orientations of the bodies exhibit planar in-phase oscillations. In both cases, the two bodies behave like a single rigid body. When Re is further increased, the configuration becomes flexible, the relative distance and relative inclination of the bodies fluctuate in time and the inclined periodic motion becomes irregular.

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1. Introduction

Non-rectilinear trajectories of solid particles, bubbles and drops moving in a fluid under the action of buoyancy occur in many natural and industrial situations involving dispersed two-phase flows. For bodies rising or falling in a fluid otherwise at rest at intermediate Reynolds numbers, significant progress has been made in the last decade concerning the understanding of the non-rectilinear paths followed by bubbles (Mougin and Magnaudet, 2006), spheres (Jenny et al., 2004; Horowitz and Williamson, 2010a), long cylinders (Horowitz and Williamson, 2010b), thin plates (Andersen et al., 2005) and short-length cylinders and disks (Fernandes et al., 2007). The path followed by a body is the result of a complex coupling between its motion and the flow it induces in the surrounding fluid. This coupling is governed by the hydrodynamic loads acting on the body, which include in particular wake-induced loads and added-mass effects. Both are strongly influenced by the geometry of the body and major kinematic differences occur when the level of anisotropy of the body is significantly changed (Ern et al., 2007; Fernandes et al., 2008). The bifurcation sequence of the wake instability of fixed three-dimensional bodies, which depends strongly on the body shape (Fabre et al., 2008), is also a crucial ingredient for the selection of the paths followed by freely moving bodies of contrasted shape. In many situations though, the motion...
of the body is coupled to the movement of the surrounding fluid, which may in particular be induced by the presence of other freely moving bodies. This flow induces in turn a modification of the hydrodynamical loads acting on neighbouring bodies, leading to different interaction behaviours, such as attraction, repulsion or synchronized oscillatory motions. These rearrangement mechanisms play an important role in fluidization and sedimentation since they lead to aggregates or clusters and reciprocally also to voidage regions (Joseph et al., 1987). Again, wake effects are known to play a major role in modifying the relative position of the bodies, in particular at low Reynolds numbers. A first interaction mechanism associated with the wake is the capture and entrainment of a trailing body by the wake of a leading body, a well-known cooperative motion also observed in animal locomotion. Fortes et al. (1987) investigated the coupled motion of spheres whose line of centres is parallel to the stream (spheres in tandem). They observed a dynamic process they called “drafting, kissing and tumbling”. The trailing sphere is drafted into the wake of the leading sphere until they kiss. At this stage, their centres are still parallel to the stream but the configuration is unstable to torques of the type which turn streamlined bodies broadside on. At variance with the drafting, this effect is related to the fluid pressure on the body and is recovered from potential flow theory (Lamb, 1932). Note that for particles falling in confined geometries, the trailing particle may turn around the leading one until it takes the lead and the process may repeat due to wall effects (Feng et al., 1994). A second interaction mechanism related to the wake is its ability to keep together particles and to build “stable architectures”, as called by Joseph et al. (1987), such as a series of spheres aligned perpendicular to the stream. However, the most remarkable stable formations occur for non-spherical bodies, like cylinders or disks (see Fig. 2). A T-configuration, in which the axes of symmetry of the bodies are perpendicular, was observed experimentally by Joseph et al. (1987) for cylinders falling between parallel plates in a two-dimensional geometry and occasionally for disks with \( \chi \leq 2 \) in a three-dimensional cell. A stable Y-configuration, in which the axes of the bodies form an acute angle, is mentioned by Joseph (1993) for long particles in the two-dimensional case. The observation of such stable doublets is also mentioned by Jayaweera and Mason (1965) for a pair of disks in a non-confined geometry. The existence of these configurations has to be associated with the fact that the wakes of fixed neighbouring bodies merge when the bodies are close enough (Williamson, 1985; Schouveiler et al., 2004). For bodies free to move, the coupling effect of the wake can be strong enough to prevent separation of the bodies, so that the configuration is retained despite its motion. However, to our knowledge, a quantitative characterization of the path dynamics of such configurations is still lacking. In this paper, we present experimental results concerning the characteristics of three different paths followed by stable doublets when the Reynolds number is increased.

2. Experiments

We carried out an experimental study on the interaction of two identical freely falling disks (diameter \( d \) and thickness \( h \)) released consecutively at two nearby locations. Three different aspect ratio \( \chi = d/h \) were used (3, 6 and 10) since important kinematic differences occur when the body anisotropy is changed (Fernandes et al., 2007). The bodies were released at the top of a tank 1.70 m high, having a square section with 40 cm sides, filled up with a homogeneous solution of salted water of kinematic viscosity \( v \). The bodies were released consecutively through a 20-cm long tube of diameter about twice to thrice that of the bodies, which prevents them from separating horizontally while they accelerate from rest. Two different types of release conditions were used. To investigate the effect of wake entrainment and the formation of doublets, the bodies were released at separate times so that their separation immediately after leaving the tube was about 18 (10) diameters for \( \chi = 3 \) (10) to match the field of observation of the cameras. To investigate the stability of the coupled-bodies configuration and its wake, the bodies were released together. For practical reasons, the density ratio between the disks and the fluid was fixed close to unity. The vertical mean velocity \( V_z \) of the bodies varied between 10 and 35 mm/s, and the corresponding Reynolds number, \( Re = Vzd/v \), varied between 80 and 300.

The fall of the bodies was followed by two perpendicular travelling cameras whose position was recorded by a high-accuracy magnetic encoder. Image processing was then used to determine the translation and orientation of the bodies via contour detection with an accuracy of \( \pm 0.06 \) mm. The image processing technique was presented in detail in Fernandes et al. (2007). In the case investigated here, however, when the vertical positions of the bodies are close, a partial occultation of a body may occur and a single contour corresponding to the two bodies is then detected (see Figs. 2 and 3). In order to dissociate the bodies, a special routine was implemented to detect the angles of the contour, based on the vectorial and scalar products of two neighbouring vectors tangent to the contour. This method allows us to separate the contour into two sets of points, each belonging to the projection of a body. Depending on the number of angles of each group, the whole contour of the body is then reconstructed. If three or four angles are visible for a body, the points are successfully fitted with an ellipse (Fitzgibbon et al., 1999), which provides the centre of gravity as well as the orientation of the corresponding body. If only two angles of a body are detected, the ellipsoidal fit is no longer suitable and the perpendicular bisector of the corresponding segment is determined. Its inclination then gives the orientation of the body, and the centre of gravity is located along the bisector at half the diameter of the body. If less than one angle of a body is detected, we consider that not enough information is available to determine properly the body position and orientation on this image. The position and orientation signals for the missing points are then interpolated using neighbouring data. Since both cameras provide a determination of the vertical coordinates of the centres of gravity of the bodies, this redundant information is used to check the accuracy of the detection, which is \( \pm 0.2 \) mm.
3. Interaction of two bodies falling in tandem

For all $\chi$ and Re investigated, we observed that the trailing body accelerates due to the wake of the leading body until it catches up with the leading body. Fig. 1 shows instantaneous pictures of the behaviour of bodies having aspect ratios 3 and 10. In order to visualize and distinguish their wakes, the bodies were covered with two different dyes (fluorescein and rhodamine) and illuminated by ultraviolet (UV) light. After the two bodies have touched, thick bodies ($\chi = 3$, for Re = 105, 145, 255 and 285) separate. Occasionally, when one body is left behind, the process may be repeated, but in all cases the two bodies eventually fall side by side. On the other hand, we observed that the wakes of thin bodies ($\chi = 6$ and $\chi = 10$) usually merge and the bodies eventually continue their fall together. For Re = 115 and 152 (respectively 105 and 125), in 27 out of 29 (21 out of 30) experiments for bodies with $\chi = 10$ ($\chi = 6$), the two bodies did not separate and stayed attached hydrodynamically. For Re = 255 and 275 (respectively 242 and 285), 100% (90%) of the pairs of bodies kept together for $\chi = 10$ ($\chi = 6$), over 30 experiments. The separation of the bodies is related to the way the trailing body enters the near

![Fig. 1. Bodies falling in tandem. Wake visualization using rhodamine and fluorescein and UV lighting. First sequence: $\chi = 3$, Re = 105, $d = 5.7$ mm and $\Delta t = 0.8$ s (0.4 s) between pictures of the first (second) row. Second sequence: $\chi = 10$, Re = 80, $d = 7$ mm and $\Delta t = 1.2$ s (0.6 s).](image)
wake of the leading body, while oscillating in position and orientation (here, for Re ≥ 152 (125) for χ = 10 (6)). No separation was observed when the path of the trailing body remained rectilinear despite the fact it accelerated. Two preferential relative positions of the bodies were observed (Fig. 2): a T-configuration, in which the symmetry axes of the bodies are perpendicular or almost so, and a Y-configuration, in which the symmetry axes form an acute angle. The T-configuration is, however, observed only occasionally, and moreover, in all cases it evolves after a few periods of oscillation into the Y-configuration, which is stable. The term “stable” is used here in the sense that the configuration is preserved over the whole length of the setup.

4. Coupled motion of thin bodies

At Re = 80 for χ = 10, which corresponds to a rectilinear path of the bodies when they fall alone, we observed that the Y-configuration falls along a rectilinear path which is slightly tilted relative to the vertical (with horizontal drift between 4% and 10%). At higher Reynolds numbers, Re = 115 and 152 for χ = 10 (105 and 125 for χ = 6) corresponding also to rectilinear paths for the single bodies, the Y-configuration displays a regular transversal oscillation in addition to the horizontal drift. Fig. 3 shows a time sequence of two perpendicular views of the falling couple. It is clear that the couple displays a periodic behaviour. Dye (permanganate potassium) was used to visualize the single wake associated with the motion of the couple: a periodic shedding of vortices in phase with the oscillatory motion is observed. The characteristics of the coupled motion of the two bodies are described in detail in the next section. It appears that the two bodies behave like a single rigid body. The centre of gravity of the pair describes a periodic path contained in a plane slightly tilted relative to the vertical and the orientations of the bodies exhibit planar in-phase oscillations. For these rectilinear and periodic paths, the bodies appear to be in physical contact, within image accuracy. At higher Reynolds numbers, Re = 255 and 275 for χ = 10 (242 and 285 for χ = 6), the bodies display a periodic zigzag path when they fall alone. When two identical thin bodies are falling in tandem at these Re, the entrainment process also leads to a Y-configuration and the bodies are still kept together by a single wake. However, at variance with the previous cases where the bodies seem to be in physical contact, the relative distance and relative inclination of the bodies fluctuate in time, so that the bodies may even adopt temporarily a T-configuration and no longer behave like a single rigid body. In this regime, the Y-configuration is thus flexible and the associated path is oblique and periodic but of irregular amplitude.

Fig. 2. Left: Two perpendicular views of the T-configuration. Right: Illustration of the Y-configuration: a lateral view and a three-dimensional reconstruction using two perpendicular cameras (χ = 10, d = 11 mm).

Fig. 3. Dye-visualization of the unsteady wake associated with the oscillatory Y-configuration (two perpendicular views; Δt = 0.4 s between pictures: (a) t = 0, (b) t = T/10, (c) t = T/5, (d) t = 3T/10, (e) t = 2T/5, (f) t = T/2, T being the period of the oscillation; χ = 10, Re = 115, d = 9 mm, U₀ = 14.9 mm/s).
4.1. Oscillatory motion

A consequence of the Y-configuration is that the two bodies fall more rapidly together (typically 15% faster) than if they were falling separately. We denote by $D_z$ the relative distance between the centres of gravity of the bodies along the vertical direction and $D_h$ along the horizontal. As shown in Fig. 4(a), $D_z$ and $D_h$ are almost constant in time, indicating that the upper body remains at an almost constant distance from the lower body.

Fig. 4(b) shows a typical time evolution of the inclinations of the bodies, i.e. of the angles of their symmetry axes with the vertical. Projected in the principal plane of the oscillations, the inclinations $\theta_x$ of the bodies display the same periodic evolution, with equal phases and amplitudes (solid lines). In the vertical plane perpendicular to the principal plane of the oscillations, the measured angles $\theta_y$ are constant in time (dashed lines). In this plane, the relative inclination $\alpha$ (see Fig. 2) between the bodies is thus constant in time. As neither the relative distance nor the relative inclination of the bodies varies in time, the couple can thus be considered to behave like a single rigid body. Fig. 5(a) displays the time evolutions of the horizontal positions of the centres of gravity of the bodies, using the same nomenclature. A slight difference in the amplitudes and phases of the horizontal displacements of the bodies is observed. This indicates that the equivalent single rigid body exhibits a yawing motion (along a vertical axis passing through its centre of gravity) in addition to the rolling motion described previously. Both rotations have the same time period and are in-phase, but the rolling amplitude is about three times larger than the yawing one.

The time evolution of the angle between an arbitrary horizontal X-axis and the principal direction of the horizontal oscillations of each body, denoted $\theta_{p1}$ and $\theta_{p2}$, is presented in Fig. 5(b). It is clear that the bodies oscillate about the same...
principal direction, which in this case rotates slightly, probably due to perturbations in the surrounding liquid. The figure also displays the time evolution of the angle $\theta_a$ between the $X$-axis and the horizontal projection of the axis passing through the centres of gravity of the bodies. We can see that $\theta_a$ oscillates weakly about a direction nearly perpendicular to the plane of the principal oscillations of the bodies, illustrating the yawing motion of the couple.

The Strouhal number of the oscillatory motion of the couple is about $St_C \approx 0.15$ for both aspect ratios $\chi = 6$ and $\chi = 10$. This value is of the order of the $St$ observed when thick disks ($\chi \leq 6$) and plates are falling alone, while a freely falling thin disk with $\chi = 10$ displays an oscillatory path at $St \approx 0.25$ (Fernandes et al., 2007). As shown in Fig. 6, the amplitudes of the rolling motion are also relatively large considering the Reynolds numbers involved. For a body with $\chi = 6$ falling alone, these values of amplitude would only be observed for Reynolds numbers about 1.5 to 2 times larger.

4.2. Horizontal drift

We have plotted in Fig. 5(b), for each body, the angles $\theta_{d1}$ and $\theta_{d2}$ between the horizontal projection of its velocity and the $X$-axis (dashed lines). We can see that the direction of the drift is perpendicular to the principal direction of oscillations, $\theta_{p1}$ and $\theta_{p2}$, and is very close to the mean direction of the axis joining the centres of gravity of the bodies, $\theta_a$. For instance, in Fig. 2, the bodies drift horizontally towards the right. We have seen in Fig. 4(b) that the relative inclination $\alpha$ between the bodies assumes a constant value as a function of time. However, this value was observed to vary from one experiment to another, within the range 20–40°, for the same given Reynolds number. As displayed Fig. 7(a), the horizontal drift varies accordingly (between 4% and 10%). Moreover, Fig. 7(b) indicates that the increase in $\alpha$ is associated with a

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**Fig. 6.** Amplitudes of the oscillatory motion of the Y-configuration for bodies with $\chi = 6$ and 10. Left: inclinations of the bodies $\theta_x$. Right : horizontal displacements $x/d$. The lines correspond to the values measured for freely falling single bodies with $\chi = 3, 6$ and 10 (Fernandes et al., 2007).

**Fig. 7.** (a) Relative inclination of the bodies, $\alpha$, versus horizontal drift. (b) Constant inclination of the bodies $\theta_y$ versus horizontal drift; $\chi = 10, Re = 115, d=9$ mm.
larger inclination of both bodies. For a given Reynolds number, a range of stable and more or less flat Y-configurations can therefore be observed.

5. Conclusion

We have shown the existence of stable configurations resulting from the interaction of two identical disks falling in tandem in a non-confined geometry. The trailing disk accelerates until it catches up with the leading body. Then, the wakes of thin disks ($\chi \geq 6$) merge in a single wake and the bodies continue their fall together adopting a stable Y-configuration, whereas thick disks ($\chi = 3$) eventually separate and fall side by side. Three-dimensional measurements of the motion of the bodies were performed to determine the characteristics of the paths associated with the Y-configuration, when the Reynolds number Re is increased. Three regimes were identified. A rectilinear non-vertical path is first observed, the horizontal drift being proportional to the inclinations of the bodies. At higher Re, the bodies exhibit in-phase oscillations in the direction perpendicular to the drift. A periodic shedding of vortices is associated with this regime. In both cases, the pair behaves like a single rigid body. When Re is further increased, the configuration becomes flexible, the relative distance and relative inclination of the bodies fluctuate along the path and the periodic motion becomes irregular. This regime is probably associated with a more complex shedding of vortices, but the gathering effect of the wake is still strong enough to prevent the separation of the disks.

References