

The 3D Transition to Turbulence in a Circular Cylinder Wake by Means of Direct Numerical Simulation (18pt, Times)

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Abstract. (11pt, times new roman, bold) The three-dimensional transition to turbulence in the flow around a circular cylinder has been analysed physically by performing direct numerical simulation to solve the system of Navier-Stokes equations. The successive stages of 3D transition, beyond the first bifurcation, have been detected first in the incompressible regime, for a circular cylinder configuration. The generation of streamwise vorticity, organised according to spanwise periodic cells has been associated with the development of large-scale coherent spanwise undulations of the originally rectilinear (nominally 2D) alternating vortex rows. The wavelengths of these undulations have been determined *versus* Reynolds number. As this parameter increases, a further inherent change of the flow transition is obtained and analysed, the natural vortex dislocations pattern. Beyond this change, the increase of Reynolds number yields an abrupt shortening of the spanwise wavelength and the flow undergoes another transition step, whose critical Reynolds number is evaluated by the present DNS approach in association with the Ginzburg–Landau model. Therefore, the linear and non-linear parts of the flow transition have been quantified by means of the amplitude evolution *versus* time obtained by the present DNS, in conjunction with the mentioned global oscillator model.

Key words: instability, transition, DNS, bluff-bodies, wings, incompressible flow.

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1. Introduction (Times New Roman 13pt, Bold)

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Concerning (times new roman, 13pt) the flow around a circular cylinder, fascinating phenomena occur in respect to the three-dimensionality of originally 2D vortex structures, as Reynolds number increases. The present study focuses beyond the first bifurcation, that consists of amplification of the von Kármán instability and of formation of the main, alternating vortex rows. Previous studies have reported that this first step to transition is essentially two-dimensional [7, 27], following a Hopf bifurcation, also known as a Poincaré–Andronov bifurcation [33]. The critical Reynolds number (order of 47) had been accurately evaluated by the Stuart–Landau global oscillator model, on the basis of the physical experiment by Provansal et al. [34], as well as by continuation methods using the steady-state approach near the threshold [19]. The DNS approach, combined with a 2D analysis of the Navier–Stokes system, offers the possibility to dissociate those mechanisms having a purely 3D origin from those being

essentially 2D, where the ensemble of these mechanisms co-exist and interact non-linearly in the context of the physical experiment. Furthermore, the DNS approach provides the detailed time-history related to the establishment of each physical mechanism, where the time-scales of the initial stages are often rather short during an experimental study for this kind of flows. In the context of flows past bluff bodies, there are still few attempts of direct simulation of the transition to turbulence, whereas this approach has been more widely used in case of rectangular configurations, including among other, boundary layer, jet and channel flows. By performing three-dimensional simulations of the flow past the cylinder in the Reynolds number range (100–300) it has been quantified that this flow pattern remains essentially two-dimensional up to Reynolds number of order 180 as shown by Persillon and Braza [28], [30]. etc..., etc... of transition are analysed in detail, in the present study.

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2. 3D Transition Features in Incompressible Wakes

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The Navier–Stokes equations for an incompressible viscous fluid past a circular cylinder are solved in a general curvilinear coordinates system normalised by the cylinder's diameter D and the uniform upstream velocity. A detailed presentation of these equations and of the numerical method developed can be found [30]. The governing equations are written in general curvilinear coordinates in the (x,y) plan while the z -component (in the spanwise direction) is in Cartesian coordinates, as presented in section 3.2.1. The numerical method is based on the 3D full Navier–Stokes equations for an incompressible fluid. The pressure-velocity formulation is used as well as a predictor-corrector pressure scheme of the same kind as the one reported by Amsden and Harlow [4], extended in the case of an implicit formulation by Braza [8] and [9], Braza et al. [7, 12]. The temporal discretisation is done by adopting the Douglas [13] fractional scheme in an Alternating Direction Implicit formulation. The method is second-order accurate in time and space. Centred differences are used for the space discretisation. The staggered grids by Harlow and Welch [17] are employed for the velocity and pressure variables. The Navier–Stokes equations are transformed in respect to a non-orthogonal, general curvilinear coordinates system in (x,y) plan, while a Cartesian coordinate z is used for the spanwise direction. This ensures the ability and general character of the present solver to take into account any complex body configuration of constant z section. An H-type grid is used because this kind of grid offers the possibility to introduce more physical boundary conditions on the external boundaries and it avoids branch-cut lines. A zoom of the grid around the obstacle is shown in Figure 1. An original aspect of the present methodology is the extension of the Douglas alternating direction fractional step scheme, initially conceived for a pure diffusion equation, to the complete set of Navier–Stokes equations. The choice of this scheme, instead of the Peaceman and Rachford [26] Alternating Direction

Implicit one, that was chosen in a previous 2D study [7], is made due to the high stability properties offered by the Douglas [13] scheme for the 3D problem. The principles of the numerical algorithm ICARE are based on the reports by Braza [10, 11]. Another useful element of the present numerical method is the extension in 3D of non-reflecting type outlet boundary conditions, based on the work by Jin and Braza [20] in two dimensions. The boundary conditions are those specified in Persillon and Braza [30] and are summarised in Figure 2. Concerning the spanwise free edges of the computational domain, periodic boundary conditions are applied. A comparison of Neumann type boundary conditions and of periodic ones in respect to their ability in simulating the development of three-dimensionality and of spanwise undulations has been performed in a number of our studies and proven their equal validity [32]. In a first simulation, Neumann boundary conditions are employed and allowed to assess the preferential spanwise wavelength formation, selected spontaneously by the flow system. However, this procedure has needed a very long transient time. Therefore, by taking a spanwise length as a multiple of the simulated one and by using periodic boundary conditions, the transient phase has been considerably shortened. **Remind: First line of a new paragraph starts at 6mm**

In the following sections the results analyse the way the 3D motion is progressively installed in the flow system. The flow evolution in time and space, with emphasis to the spanwise direction, is studied by performing appropriate signal processing techniques (wavelet analysis and autoregressive modelling). The numerical simulations are carried out for a rather long spanwise length value (12D) and part of the instability analysis concerning global oscillator models have been carried out by using a spanwise length of 4.5D. The typical grids are of order (250 * 100 * 80) to (280 * 120 * 100). The parallel computers used are the IBM-SP2 of CNUSC (Centre National Universitaire Sud de Calcul) and of Cornell Theory Centre, as well as the CRAY C98 and T3E of the national computing centre IDRIS. In the following, x is the direction along the cylinder's downstream axis, parallel to the free-stream u velocity. y is the vertical direction parallel to v velocity and z is the spanwise direction, parallel to w velocity component

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2.1. ONSET OF THREE-DIMENSIONALITY (First letter 12pt, capitals, Times new Roman, 11pt)

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The present elliptic flow loses the memory of the initial conditions applied, after a transient phase whose time scale depends on the nature of the initial conditions. After detailed numerical tests in our research group [3, 32], it has been shown that the final, established stage, after the transient one is independent on the initial conditions. Therefore, the choice of the initial fields has been done on the criterion of shortening the transient phase. The initial conditions are those of a 2D established flow with vortex shedding, providing a rectilinear configuration of the alternating vortex rows along the span. A special attention is devoted to

ensure that *the alternating vortex shedding is extended over the whole downstream distance*, and that the wake expansion rate along x is correct, according to the physical experiment. These features have been achieved by ensuring the three following conditions: (1) A sufficiently refined grid along the whole downstream distance and not only in a limited distance around the obstacle; (2) A sufficiently large computational domain in x and y directions, in order to not confine the flow and to allow the correct wake expansion; (3) Adequate boundary conditions in the downstream outlet boundary, that limit the feedback effects and allow travelling the alternating eddies through this fictitious boundary without confinement (Figure 3a). Therefore, the proper development of the primary instability, the von Kármán mode, is ensured along the whole computational domain (Figure 3a). This is the fundamental characteristic of this category of flows and a prerequisite to study further on the development of secondary instabilities along the span. In a number of related studies, the alternating character of the vortex shedding is limited within only one or two diameters downstream the obstacle.

After a long transient phase, needing more than 300,000 time steps during which the flow remains 2D, the w velocity component (in the spanwise direction) is progressively developed. In order to shorten this phase, a w velocity weak intensity fluctuation is applied in the inlet section along the span. A white noise of dimensionless intensity 10^{-4} is chosen. Different runs with different values of this intensity within the range $(10^{-4}, 10^{-3})$ have been checked and ensured leading to the same final established stage. A w velocity component typical evolution is shown in Figure 15a. The amplification of this component follows two distinct stages before saturation, a first exponential growth (linear state) and a second, non-linear one, before reaching saturation, as it is also discussed in Section 2.3. It is found that w velocity component is soon organised according to well distinct cells, (Figure 3b), already in the very early stages of onset of the secondary instability, where the w magnitudes are still very weak. As it is therefore expected from the continuity equation for an incompressible fluid, this pattern is followed by appearance of organisation of streamwise vorticity component ω_x (Figure 3c) along the span, as it is shown at time $t=780$. The birth of streamwise vorticity ω_x is due to the progressive development of w component as Reynolds number increases, owing to the influence of small perturbations mentioned before. Due to the action of consequent small longitudinal perturbations, the selected mode by the present system provided by the direct simulation is the organisation of the streamwise vorticity pattern on counter-rotating vortices. This is in accordance to the elliptic instability theory. It is recalled that the stability of an elliptical vortex configuration (as one of the von Kármán vortices in this case) to small 3D perturbations provides counter-clockwise longitudinal vorticity filaments with selected wavelengths, that can be assessed by elliptic stability theory considerations [39]. Under the effect of the progressive increase of the streamwise vorticity, the 2D main alternating vortices display a weak regular spanwise undulation. This starting 3D modification of the originally rectilinear

alternating vortices is shown in Figure 3d. At higher time values, the streamwise vortices become more intense (Figure 3e) and inception of them occurs between two main alternating eddies in the formation region (Figure 3f). Afterwards, the streamwise vortices are displaced towards the wake's shear layer, in the convection region. Figures 4a-4c show the progressive development of mode A in conjunction with the streamwise vortices formation. The intensity of the spanwise undulation increases from time $t=680$ to $t=740$ under the simultaneous effect of streamwise vorticity that is strengthened and forms progressively counter-clockwise 'braid' like structures (see pairs of red and yellow vortices), Figures 4a-4d.

The undulation of mode A, that appears as an inherent characteristic of the flow obtained by the direct numerical simulation, is clearly visible on the spanwise evolution of the vertical velocity component v , (Figure 5), that displays a spanwise oscillation according to a regular wavelength. A spectral analysis of these evolutions is performed by means of Fast Fourier Transform Figure 6, to quantify mode A wavelength. The most predominant spatial mode is found 0.1171, yielding a wavelength value $\lambda_z/R=8.54$ ($\lambda_z/D=4.27$). This value is found in agreement with the range provided by experimental results, as reported by Persillon and Braza [30]. In the present DNS results, there exist phases of the flow where mode A pattern is less regular (see, for example, the iso-vorticity contours in the time interval (800,840)). This *irregularity* is associated and *announces* a further fundamental modification of the spanwise structure of the main, alternating vortex rows, as it will be analysed in the next section.

2.2. THE NATURAL VORTEX DISLOCATIONS PATTERN

The instantaneous iso-vorticity fields ω_z and ω_x are first considered. During the time interval (800, 820) (Figures 4d and 4e), a noticeable change in the braid configuration of the red and yellow spanwise vortices occurs, that progressively aspirates fluid from the first violet main eddy and simultaneously this braid is displaced in a lower position. Under this effect, a clear discontinuity is obtained along the core of the second main violet vortex (Figure 4f). The braid of streamwise vortices *loses its spatial coherence* during the time interval (800–820), Figures 4e and 4f and at the same time, the ensemble of the streamwise vortex structures is much more fragmented even in near wake positions. During these phases of the flow, mode A is also less regular.

This fundamental modification occurring on the spanwise structure of the already undulated main vortex rows *in the near wake* is called a *natural vortex dislocation*. In the present study the vortex dislocations are obtained naturally by means of the complete system of equations, offered by the Navier–Stokes approach and are found in very good agreement (Figure 7b) with the experiment, for Reynolds number 220. Summarising, it can be stated that a vortex dislocation is a *local break of continuity* appearing on 'the spinal column' of a main vortex

row, previously subjected to a regular spanwise undulation. This 3D modification of the von Kármán vortex row appears as a local junction with the previous alternating vortex row rotating in the same sign. At phases of the flow following the development of vortex dislocations it can be seen that the braid structure of streamwise vorticity almost disappears on the profit of more fragmented streamwise structures of even smaller vortices. In these intervals it can be seen that the waviness of the main vortices becomes even more fragmented along the span. It displays progressively a $2\lambda_z$ predominant wavelength, as it will be quantified in a next paragraph. The coherence of this wavelength doubling is a consequence of the perturbation caused by the passage of the vortex dislocation.

The fact that the dislocation pattern obtained by the present DNS is accompanied by a loss of regularity of the streamwise vortex structures and of mode A undulation is shown, beyond the iso-contours of flow quantities, on the velocity components signals along the span and *versus* time, Figure 8. The marked positions 1 to 3 indicate the spanwise region where vortex dislocations are developed. It is shown that w component presents at these positions a considerable *amplitude increase* and loses its periodic character during the time intervals corresponding to the formation of vortex dislocations. Amplitude and frequency modulations are also developed on the v velocity signals. Therefore, it is obtained that the dislocation phenomenon is indeed associated with large-scale velocity fluctuation irregularities that had been observed by Roshko [36] in this category of flows and were linked to the onset of 3D effects. From the discussed overall flow evolution, it can be seen that, due to the vortex dislocations, the occurrence of mode A has an intermittent character, that explains also the dispersion of experimental wavelength values.

The contour plots of the vortex structures in the physical space presented in the beginning of the discussion, have shown that the passage of the dislocation is locally associated with the occurrence of a '*number of events-1*' in the time evolution of the vortex shedding. Therefore, a local frequency decrease is expected to occur. It is therefore worthy to quantify the frequency and amplitude variations *versus* time, during the passage of a vortex dislocation. This is done by performing autoregressive modelling signal processing and wavelet analysis applied on the signals issued by the present direct simulation. In the following figures, the dimensionless frequency is normalised in respect to the cylinder's radius. etc..., etc...

Therefore, by means of the present analysis, the tendency of the flow system to reduce its fundamental frequency is clearly indicated, during specific time intervals corresponding to the formation and advection of a vortex dislocation structure. This confirms the tendency shown by experimental data that a *lower path of fundamental frequency variation* versus Reynolds number would be followed by the system if vortex dislocations occur (Figure 10). We have plotted in this figure a mean value of the obtained frequency reduction, evaluated within the mentioned time interval above. This value is very close to the experimental path of the curve.

It is interesting to provide an overview of the effect of the vortex dislocations passage in a space-time map. The overall time-dependent evolution of the spanwise flow structure can be seen in Figure 11 where iso- v surfaces are shown as a function of z and time. etc..., etc... This is followed by formation of the vortex dislocation affecting considerably the Strouhal number; in fact the decrease of the fundamental is visible in the evolution of the fronts in the interval (1050–1070). During this phase of the motion, the perturbation of mode A propagates quasi-linearly and the propagation velocity of this event is found to follow a slope having an approximate value 0.0509 (see dashed lines on the plot). Afterwards, the dislocation occurrence seems to play a perturbing role to mode A, that is found to undergo progressively a lengthening of its wavelength towards $2\lambda_z$ value, the appearance of a first subharmonic spanwise frequency. The quantification of the predominant wavelengths along the span is confirmed by Figure 12a, where the wavenumber of the fundamental is plotted *versus* time. In Figure 12b, the longitudinal velocity component is plotted along the span in upstream and downstream positions. It is clearly shown that a velocity bump formation occurs upstream, as a selected spanwise evolution of the system, under the action of the upstream streamwise small perturbations. This preferential structure creates the local Strouhal number variation, associated with the passage of the vortex dislocations and with the *local frequency variation quantified by the present wavelet analysis*.

2.3. THE SPANWISE UNDULATION AS REYNOLDS NUMBER FURTHER INCREASES

At Reynolds number values higher than 220–230, the present DNS shows that the spanwise undulation undergoes a substantial decrease of its wavelength and the streamwise vortex filaments become more continuous, linking the formation to the convection region (Figure 13). They pass from the lower towards the upper shear layers, by materialising well distinct vortex paths. This displacement was found to be discontinuous in the case of mode A undulation (Figure 3f) [41]. Mode B constitutes an abrupt change in the flow transition [41] and it is worthy to examine whether this bifurcation is hysteretic. We have performed detailed direct simulations up to the established stage, from the upper side of Reynolds number ($Re = 340$) and descending towards $Re = 230$. The Strouhal number values according to these simulations are shown in Figure 14, [3] and indicate that this mode is characterised by hysteresis and corresponds to a values-discontinuity (and not a slope discontinuity).

The present direct numerical simulation furnishes in detail the amplitude evolution *versus* period of the flow properties, as it is shown on Figure 15a, concerning the w velocity component along the rear axis of the near wake. Based

on the amplitude results, it is possible to evaluate the real part of the coefficients of the Ginzburg–Landau global oscillator model:

Examples of writing equations:

$$\partial A/\partial t = \sigma_r A - l_r |A|^3 + \mu_r \partial^2 A/\partial z^2. \quad (1)$$

The term $\mu_r \partial^2 A/\partial z^2$ can also be expressed through a spanwise wavenumber, $\alpha=2\pi/\lambda_z$ in respect to the vortex rows undulation of wavelength λ_z [23]:

$$\partial A/\partial t = \sigma_r A - l_r |A|^3 + \mu_r \alpha^2 |A|. \quad (2)$$

The first term on the right - hand side represents the linear growth, the second term corresponds to the non-linear effects before saturation and the third term represents the 3D spanwise undulation, under the form of a diffusion formulation. A is the amplitude evolution *versus* period.

The exponential growth coefficient σ_r can be written as a function of the critical Reynolds number near the threshold:

$$\sigma_r = k (Re - Re_{cr}). \quad (3)$$

According to Provansal et al. [34], k represents a reduced frequency and σ_r can be also written as:

$$\sigma_r = (1/5) (v/D^2) (Re - Re_{cr}). \quad (4)$$

where v is the cinematic viscosity and D the cylinder's radius.

If this model is verified by the physics of the present bifurcation, σ_r must therefore vary *linearly* as a function of Reynolds number near the threshold. This is examined by the present direct simulation, and discussed in more details by Allain [3]. The amplitude evolution of Figure 15a is plotted in logarithmic scale (Figure 15b). It is clearly obtained that the first stage corresponds *indeed to a linear growth*. The slope of this curve provides the value of the σ_r coefficient. By discretising afterwards the 2D part of the amplitude evolution (e.g. corresponding to the Stuart–Landau model) in the same way as in Section 3.2, the non-linear growth coefficient l_r is evaluated (Figure 15c) from the amplitude evolution corresponding to a time - scale prior to onset of significant 3D effects. This has been verified by flow quantities visualisation. The sign of l_r coefficient indicates that mode B instability is supercritical, and this has been proven in the present study by the fully non-linear approach provided by the direct simulation.

Three complete direct simulation studies have been performed, at Reynolds numbers 280, 300 and 340, for each of them σ_r coefficient has been evaluated. Figure 15d shows σ_r *versus* Reynolds number and the best-fitted curve passing among these points. etc..., etc...

The evaluation of σ_r and l_r coefficients allows assessment of the real part of the Ginzburg–Landau coefficient, μ_r in respect to the 3D growth. In the saturation stage, the term $\partial A/\partial t$ vanishes. Therefore,

$$\mu_r = (\sigma_r - l_r A^2) / \alpha^2. \quad (5)$$

α can be evaluated by the direct simulation data for mode B, concerning λ_z , found 1.28 for Reynolds numbers (280, 300, 340). This yields an assessment of dimensionless μ_r values: ($1.3 * 10^{-3}$, $1.5 * 10^{-3}$, $2.2 * 10^{-3}$). Experimental results by Albarède and Provansal [1] provide dimensional μ_r values of order 10ν to 40ν , that yields dimensionless values of order $0.2 * 10^{-2}$ to $5 * 10^{-2}$, for a lower Reynolds number range, and corresponding to chevron patterns due to non-parallel shedding conditions.

3. Conclusions

The present study is a continuation of our works in the domain of analysis of 3D transition in flows around bluff bodies. A detailed investigation of successive steps of transition is carried out, as Reynolds and Mach number increase. A special emphasis is devoted to the analysis of natural vortex dislocations in the low Reynolds number range. It has been shown that these structures are a *systematic and inherent transition feature*, modifying drastically the first 3D step to transition, that consists of the development of a regular spanwise undulation. These transition characteristics are predictable by the fully non-linear approach offered by the direct simulation.

The present study has *quantified in detail the frequency reduction* associated with the formation of this kind of vortex structures, by means of autoregressive signal processing and wavelet analysis of the time-dependent velocity and vorticity signals along the span. spanwise location of the dislocation. This feature is obtained in the time and frequency domains concerning the predominant amplitude of the fundamental frequency.

The interaction of the vortex dislocations with mode A is shown in the present study as a perturbing effect that modifies the regular spanwise undulation formation. etc..., etc... after *a long time scale*, including an order of 20 periods of the vortex shedding is the enhancement of *the first subharmonic wavelength* of mode A, as the most energetic mode.

The successive transition step to dislocations mode undergoes a significant shortening of the spanwise wavelength and a modification on the streamwise vortex filaments, in accordance with the physical experiments concerning mode B. By employing the present DNS results, the supercritical nature of this instability has been shown. This successive step of transition is able to be modelled by the Ginzburg–Landau equation, whose real-part coefficients are evaluated on the basis of the present direct simulation results.

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Acknowledgements

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This work has been carried out in the research group EMT2 (Ecoulements Monophasiques, Transitionnels et Turbulents) of the Institut de Mécanique des Fluides de Toulouse. It is based on the collaborative efforts of J. Allain, A. Bouhadji, D. Faghani and H. Persillon in the context of their Ph.D theses and post-doctoral research. Part of this work is carried out on the basis of CPU allocations of the two national computer centres of France CNUSC and IDRIS, as well of the Cornell's Supercomputing centre.

Figures can be put either in the text or at the end of the text, as in the exemple (11pts) ; (it is also possible to put two figures in the page's width)

Examples for figure captions:

Figure 1. Zoom of the grid around the body and in the near wake. (Times new Roman, 11pt)

Remark: Figure captions start flush left below the figure, at 1,5 cm from the figure's frame.

Figure 2. Computational domain and boundary conditions.

Table captions are put above the Table, at 1cm from the Table's frame.

Figure 3a. Iso-vorticity contours ω_z showing persistence of the *alternating* vortex pattern over the whole computational domain

Examples for figure position and legend:



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Figure 3. (e): Development of longitudinal vorticity cells between the main eddies in the formation region. (f): Displacement of the longitudinal vortices in the shear layers, in the convection region. Same graduation as in previous figure.

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